Health Risk and the Value of Life*

Daniel Bauer
University of Wisconsin-Madison

Darius Lakdawalla
University of Southern California
and NBER

Julian Reif
University of Illinois and NBER

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Abstract

We develop a stochastic life-cycle framework for valuing health and longevity improvements and apply it to data on mortality, quality of life, and medical spending for adults with different comorbidities. Contrary to conventional theory, we find that sick adults are willing to pay over twice as much per quality-adjusted life-year (QALY) to reduce mortality risk than healthy adults, and that prevention of serious illness risk is worth more per QALY than prevention of mild illness risk. Our results provide a rational explanation for why people oppose a single threshold value for rationing care and why they invest less in prevention than in treatment.

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1 Introduction

The economic analysis of risks to life and health has made enormous contributions to academic discussions and public policy. Economists have used the standard tools of life-cycle consumption theory to propose a transparent framework that measures the value of improvements to both health and longevity (Murphy and Topel, 2006). Economic concepts such as the value of statistical life (VSL) play central roles in discussions surrounding public and private investments in medical care, public safety, environmental hazards, and countless other arenas.

However, the conventional life-cycle framework used to study the value of life includes only a single health state, with a preordained mortality rate that depends on age alone. As a result, it is ill-equipped to investigate how VSL varies with underlying health or with health shocks, and it cannot distinguish between preventive care and medical treatment or between illness and death. These shortcomings greatly limit its explanatory power and policy relevance. For example, an array of evidence suggests that society invests less in prevention than treatment, even when both have the same health benefits (Weisbrod, 1991; Dranove, 1998; Pryor and Volpp, 2018). The conventional framework's failure to explain this apparent underinvestment in prevention has led researchers to posit alternative behavioral or market failure explanations, although the evidence remains inconclusive (Fang and Wang, 2015; Bai et al., 2021; Newhouse, 2021).

Likewise, the conventional model's reliance on a single health state hampers its ability to engage in the ongoing policy debates regarding whether and how healthcare reimbursements should vary with health shocks of differing severity. In many countries, a medical treatment is covered by insurance only if its price meets a "cost-effective" threshold, and the underlying theory implies that this threshold should not vary with patient health or with the severity of the illness being treated (Hammitt, 2013; Lucarelli et al., 2022).[1] However, survey research finds scant public support for a constant threshold (Nord et al., 1995; Linley and Hughes, 2013), and several countries that rely on cost-effectiveness to allocate healthcare resources—including Norway, Sweden, the Netherlands, and the UK—have defied the underlying economic theory by codifying *ad hoc* approaches that increase the cost-effectiveness threshold for treatments of severe diseases.[2]

[1] US law prohibits federal agencies such as Medicare from using conventional cost-effectiveness methods (Lakdawalla and Phelps, 2022).

[2] Norway and Sweden identify disease severity as a core determinant of treatment value (Defechereux et al., 2012; Persson et al., 2012). The Netherlands relies on "proportional shortfall" methods that increase reimbursements for treatments of diseases that cause greater relative reductions in quality-adjusted life expectancy (Reckers-Droog et al., 2018). The UK has recently formalized similar *ad hoc* approaches in its latest health technology evaluation manual (NICE, 2022).

This paper studies how a rational individual's marginal value of reducing mortality and illness risk varies with baseline health status and with the severity of those risks.[3] We develop a new stochastic life-cycle framework that accommodates multiple health states with different quality of life and mortality risk profiles. We derive the value of statistical illness (VSI), which measures the willingness to pay to reduce illness risk and includes VSL as the special case where that risk is death. We focus initially on a setting without insurance markets, and then later extend our results to a more realistic setting with incomplete insurance markets. We obtain two important insights, one regarding how the value of a given health risk reduction varies across people in different baseline health states and another regarding how a given individual's value of a health risk reduction varies with the severity of that health risk. First, willingness to pay for an additional unit of health is higher for individuals with shorter baseline life expectancy under standard risk assumptions. Second, prevention of serious illness risk is worth more per health unit than prevention of mild illness risk. These findings contrast with both conventional VSL theory, which does not accommodate multiple health states, and with conventional cost-effectiveness theory, which posits that the value of a health unit does not vary with current health status or with the severity of a prospective illness risk.

The first theoretical insight described above, which compares the value of life across individuals with different life expectancies, hinges on two countervailing effects. Consider an individual newly diagnosed with cancer. On the one hand, the attendant reduction in her life expectancy reduces VSL by reducing lifetime utility. This same channel drives the well-known result that VSL declines at older ages. On the other hand, this unexpected reduction in longevity increases VSL by encouraging her to spend down her wealth more quickly. This second effect cannot occur in a conventional model with only one health state, because stochastic reductions in longevity cannot happen when the health state is permanently fixed. To clarify when one effect dominates the other, we derive a sufficient condition for VSL to rise following an adverse shock to longevity. This condition, which depends on how prudence (Kimball, 1990) compares to the elasticity of intertemporal substitution, is satisfied by many standard utility functions as well as current estimates of those two parameters.

Next, we consider how the value of a unit of health—which is the policy-relevant measure for countries using cost-effectiveness—varies with the individual's baseline health state. When VSL rises following a reduction in longevity, the value of a unit of health

[3]In the remainder of the paper, "health" refers to both longevity and health-related quality of life. We use the terms "health risk" and "illness risk" interchangeably. A "unit of health" measures both longevity and health-related quality of life.

necessarily rises as well. We investigate whether this result holds true in a more general setting where both quality of life and longevity vary across health states. We show that the value of a unit of health is higher for individuals in worse health, provided that individuals are risk averse over illness severity, i.e., that they prefer living with mild illness for certain to living in good health with a risk of severe illness. This condition also leads to our second theoretical insight: just as a risk-averse individual is willing to pay more per dollar to insure larger losses, an individual who is risk averse over illness severity is willing to pay more per unit of health improvement to reduce the risk of more serious illnesses.

To assess the economic significance of these two theoretical findings, we apply our model to individual-level data from a representative cohort of US adults ages 50–80. The data are obtained from the Future Elderly Model (FEM), which provides detailed information on how mortality, medical spending, and quality of life evolve over the life cycle for people over age 50 with different comorbidities. The data underlying the FEM include more comprehensive information than any single national survey and have been widely used to study elderly health and medical spending (e.g., Goldman et al., 2010, 2013; Reif et al., 2021). We quantify each individual's marginal willingness to pay for the prevention and treatment of twenty different health conditions with varying mortality, quality of life, and financial risk profiles. We measure health improvements in units of quality-adjusted life-years (QALYs), a widely used metric that combines the quality and quantity of life-years into a single index.

In line with our first theoretical result, we find that VSL rises on average by $88,000 (21%) per QALY in the year following an adverse health shock, and by over $200,000 (58%) per QALY following the worst five percent of shocks. Among 70-year-olds, those in the sickest health state are willing to pay 2.4 times more per QALY to reduce mortality risk than healthy people, representing a wide gap between the value of treatment and prevention. To assess our second theoretical result, we quantify the values of different preventive investments for a consumer in a fixed health state. We find that the value of prevention rises only modestly with disease severity: a healthy 70-year-old is willing to pay up to $36,000 (16%) more per QALY to reduce extreme risks such as serious cancer or death than to reduce mild risks such as developing hypertension. Consistent with our theory, the value of life varies less dramatically across different forms of prevention than across treatments and prevention.

We perform several sensitivity analyses to assess the robustness of our results. While the absolute values of our estimates are moderately sensitive to alternative assumptions about consumer risk preferences or the presence of a bequest motive, our two qualitative conclusions—that the value of reducing a health risk increases with baseline health and with severity of the risk—hold up across a number of alternative parameterizations. We also confirm that a health state's mortality risk profile is the key factor driving our numerical results: we obtain the same patterns if we omit medical spending from our analysis and assume quality of life remains constant across health states.

Our primary contribution is the development and application of a new, more general life-cycle model of the value of life. While an individual's value of health risk reduction in the conventional model depends on her age and wealth, in our model this value depends also on her current state of health and on the characteristics of the health risks she faces. Our finding that the value of a QALY is up to 2.4 times higher among the sick than the healthy helps explain puzzles such as why consumers invest less in prevention than treatment, why end-of-life spending is high (Zeltzer et al., 2021), why cost-effectiveness analysis appears out-of-step with policies promoting more generous reimbursement of care for more severe diseases (Lakdawalla et al., 2014), and why preventive care interventions frequently fail to improve health (Jones et al., 2019), without needing to resort to alternative explanations such as market inefficiencies or irrational behaviors. Our findings also explain consumer and voter opposition to the use of a single threshold value when making decisions about health resource allocation, and provides further support for "top-up" insurance policies, which allow patients who value health more highly to pay incremental prices for more expensive treatments (Einav et al., 2016; Lucarelli et al., 2022). With its (health) states appropriately redefined, our stochastic framework can also be applied to a number of other distinct questions, such as why societies appear to invest less in preventing pandemics than in mitigating them and how to value insurance in a setting with shocks to health, longevity, and spending (Kowalski, 2015; Ericson and Sydnor, 2018; Fang and Shephard, 2019; Atal et al., 2020). Finally, while a stochastic framework can be challenging for practitioners to use in policy analysis, we provide a closed-form solution and an accompanying open-source tool to rapidly calculate VSL and VSLY.

The economic literature on the value of life includes seminal studies by Arthur (1981), Rosen (1988), Murphy and Topel (2006), and Hall and Jones (2007). Shepard and Zeckhauser (1984) and Ehrlich (2000) note the important role played by insurance markets. Aldy and Smyth (2014) use microsimulation to assess heterogeneity in VSL by race and sex. Córdoba and Ripoll (2016) use Epstein-Zin-Weil preferences to study the implications of state non-separable utility on the value of life. The models used in these prior studies include only a single health state for living individuals and focus exclusively on the value of preventing death, setting illness aside. Our study increases the scope and relevance of standard economic theory for understanding health risk by, for example, allowing researchers to compare the value of treatment to prevention and to quantify the relative values of reducing the risk of different illnesses.[4]

Our model also reconciles the standard life-cycle framework with results from a distinct literature that uses one-period models to study the value of mortality risk-reduction (Raiffa, 1969; Weinstein et al., 1980; Pratt and Zeckhauser, 1996; Hammitt, 2000). These static models predict that an increase in baseline health risk must raise VSL when insurance markets are incomplete, a result often referred to as the "dead-anyway" effect. We show that this result does not apply to a dynamic life-cycle setting. In fact, adverse longevity shocks can raise *or* lower VSL, depending on consumer risk preferences.

The remainder of this paper is organized as follows. Section 2 presents the model, derives key results, and discusses welfare. Section 3 applies the model to data and shows how VSL varies across people with different health histories and and how the value of preventing illness varies with the degree of illness risk. Section 4 concludes.

2 Model

Consider an individual who faces a health risk such as illness or death. We are interested in analyzing the value of a marginal reduction in that risk. We begin with a "Robinson Crusoe" model where the consumer cannot incur debt or purchase annuities to insure against her uncertain longevity. This simple setting allows us to transparently communicate our main insights; we then later show how these insights extend to a more realistic setting with incomplete insurance markets.

Section 2.1 solves a Robinson Crusoe model that accommodates multiple health states. We derive VSI, the value of reducing a risk of illness or death. VSI depends on (i) the individual's current health state, which lets us investigate how the value of life varies with baseline health; and (ii) the characteristics of the health risk, which lets us compare the risk-reduction values of different kinds of illnesses. Section 2.2 provides a sufficient condition under which VSL—a special case of VSI—rises following an adverse shock to longevity. Section 2.3 describes how the value of a marginal health *unit* varies across people in different health states and with the characteristics of the health risk. Section 2.4 extends our results to an incomplete markets setting where the consumer earns income over the life-cycle, has access to health care insurance, and can optimally invest her wealth in a constant annuity. Section 2.5 discusses welfare. Because a complete markets setting lacks realism, we relegate its analysis to Appendix D.

[4] Several empirical papers have already demonstrated the relevance of valuing illness-prevention (Cameron and DeShazo, 2013; Hummels et al., 2016).

Like prior studies on the value of life, we focus throughout this paper on the demand for health and longevity. Quantifying optimal health spending requires additionally modeling the supply of health care (Hall and Jones, 2007). In light of all the institutional differences across health care delivery systems, a wide variety of plausible approaches can be taken to this modeling problem, which we leave to future research.

2.1 The value of health and longevity

Let Y_t denote the consumer's health state at time t. We assume Y_t is a continuous-time Markov chain with finite state space $Y = \{1, 2, \ldots, n, n+1\}$, where state $i \in \{1, \ldots, n\}$ represents different possible health states while alive, and state $i = n+1$ represents death. Denote the transition rates by:

$$\lambda_{ij}(t) = \lim_{h \to 0} \frac{1}{h} \mathbb{P}[Y_{t+h} = j | Y_t = i], \ j \neq i,$$
$$\lambda_{ii}(t) = -\sum_{j \neq i} \lambda_{ij}(t)$$

For analytical convenience and without meaningful loss of generality, we assume that individuals can transition only to higher-numbered states, i.e., $\lambda_{ij}(t) = 0 \ \forall j < i$.[5] The probability that a consumer in state i at time 0 remains in state i at time t is then equal to:

$$\tilde{S}(i,t) = \exp\left[-\int_0^t \sum_{j>i} \lambda_{ij}(s)\, ds\right]$$

For expositional purposes we shall refer to transitions as either "falling ill" or "dying," but our model also accommodates transitions from sick states to healthy states. We denote the stochastic mortality rate at time t as:

$$\mu(t) = \sum_{i=1}^{n} \lambda_{i,n+1}(t)\, \mathbf{1}\{Y_t = i\}$$

where $\mathbf{1}\{Y_t = i\}$ is an indicator variable equal to 1 if the individual is in state i at time t and 0 otherwise. When the number of states is equal to $n = 1$, we obtain the setting with deterministic health risk studied in prior literature (e.g., Shepard and Zeckhauser, 1984; Rosen, 1988; Murphy and Topel, 2006). The maximum lifespan of an individual is T, and

[5]That is, a person can transition from state i to j, $i < j$, but not vice versa. This restriction does not meaningfully limit the generality of our model because one can always define a new state $k > j$ with properties similar to state i.

we denote her stochastic probability of surviving until $t \leq T$ as:

$$S(t) = \exp\left[-\int_0^t \mu(s)\,ds\right]$$

Let $c(t)$ be consumption at time t, W_0 be baseline wealth, ρ be the rate of time preference, and r be the rate of interest. Health-related quality of life at time t, $q_{Y_t}(t)$, is exogenous and depends on the health state, Y_t. Let the state variable $W(t)$ represent current wealth at time t. Normalizing the utility of death to zero, the consumer's maximization problem for $Y_0 \in \{1, \ldots, n\}$ is:

$$V(0, W_0, Y_0) = \max_{c(t)} \mathbb{E}\left[\int_0^T e^{-\rho t}\, S(t)\, u(c(t), q_{Y_t}(t))\, dt \,\middle|\, Y_0, W_0\right] \tag{1}$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ \frac{\partial W(t)}{\partial t} &= rW(t) - c(t) \end{aligned}$$

The no-debt constraint, $W(t) \geq 0$, means the consumer cannot borrow. The utility function, $u(c, q)$, is time-separable and depends on both consumption and health-related quality of life. We assume throughout that $u(\cdot)$ is strictly increasing and concave in its first argument, and twice continuously differentiable. Hence, we must have $W(T) = 0$, since it cannot be optimal to have wealth remaining at the maximum possible age. We denote the marginal utility of consumption as $u_c(\cdot)$ and assume that this function diverges to positive infinity as consumption approaches zero, so that optimal consumption is always positive.

Define the consumer's objective function at time t as:

$$J(t, W(t), i) = \mathbb{E}\left[\int_0^{T-t} e^{-\rho u} \exp\left\{-\int_0^u \mu(t+s)ds\right\}\, u\left(c(t+u), q_{Y_{t+u}}(t+u)\right) du \,\middle|\, Y_t = i, W(t)\right]$$

Define the optimal value function as:

$$V(t, W(t), i) = \max_{c(s),\ s \geq t} \{J(t, W(t), i)\}$$

subject to the wealth dynamics above and $V(t, W(t), n+1) = 0$. Under conventional regularity conditions, if V and its partial derivatives are continuous, then V satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\rho V(t,W(t),i) = \max_{c(t)} \left\{ u(c(t),q_i(t)) + \frac{\partial V(t,W(t),i)}{\partial W(t)} [rW(t) - c(t)] + \frac{\partial V(t,W(t),i)}{\partial t} \right.$$
$$\left. + \sum_{j>i} \lambda_{ij}(t) [V(t,W(t),j) - V(t,W(t),i)] \right\}, \; i = 1,\ldots,n \quad (2)$$

where $c(t) = c(t,W(t),i)$ is the optimal rate of consumption.

In order to apply our value of life analysis, we exploit recent advances in the systems and control literature. Parpas and Webster (2013) show that one can reformulate a stochastic finite-horizon optimization problem as a deterministic problem that takes $V(t,W(t),j)$, $j \neq i$, as exogenous. More precisely, we focus on the path of Y that begins in state i and remains in state i until time T. We denote optimal consumption and wealth in that path by $c_i(t)$ and $W_i(t)$, respectively.[6] A key advantage of this method is that it allows us to apply the standard deterministic Pontryagin maximum principle and derive analytic expressions.

Lemma 1. *Consider the following deterministic optimization problem for $Y_0 = i$ and $W(0) = W_0$:*

$$V(0,W_0,i) = \max_{c_i(t)} \left[\int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t),q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t,W_i(t),j) \right) dt \right] \quad (3)$$

subject to:

$$W_i(0) = W_0,$$
$$W_i(t) \geq 0,$$
$$\frac{\partial W_i(t)}{\partial t} = rW_i(t) - c_i(t)$$

where $V(t,W_i(t),j)$, $j \neq i$, are taken as exogenous. Then the optimal value function, $V(t,W_i(t),i)$, satisfies the HJB equation given by (2), for all $i \in \{1,\ldots,n\}$.

Proof. See Appendix A ■

[6]Consumption, $c(t)$, is a stochastic process. We occasionally denote it as $c(t,W(t),Y_t)$ to emphasize that it depends on the states $(t,W(t),Y_t)$. When we reformulate our stochastic problem as a deterministic problem and focus on a single path $Y_t = i$, consumption is no longer stochastic because there is no uncertainty in the development of health states. We emphasize this point in our notation here by writing consumption as $c_i(t)$, and wealth as $W_i(t)$.

Because the value function $V(t, W_i(t), i)$ corresponding to (3) satisfies the HJB equation given by (2), it must also be equal to the consumer's optimal value function (Bertsekas, 2005, Proposition 3.2.1). The present value Hamiltonian corresponding to (3) is:

$$H\left(W_i(t), c_i(t), p_t^{(i)}\right) = e^{-\rho t}\tilde{S}(i,t)\left(u(c_i(t), q_i(t)) + \sum_{j>i}\lambda_{ij}(t)V(t, W_i(t), j)\right) + p_t^{(i)}\left[rW_i(t) - c_i(t)\right]$$

where $p_t^{(i)}$ is the costate variable for state i. The necessary costate equation is:

$$\dot{p}_t^{(i)} = -\frac{\partial H}{\partial W_i(t)} = -p_t^{(i)}r - e^{-\rho t}\tilde{S}(i,t)\sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \tag{4}$$

The solution to the costate equation can be obtained using the variation of the constant method:

$$p_t^{(i)} = \left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s, W_i(s), j)}{\partial W_i(s)}ds\right]e^{-rt} + \theta^{(i)}e^{-rt}$$

where $\theta^{(i)} > 0$ is a constant. The necessary first-order condition for consumption is:

$$p_t^{(i)} = e^{-\rho t}\tilde{S}(i,t)u_c(c_i(t), q_i(t)) \tag{5}$$

where the marginal utility of wealth at time $t = 0$ is $\frac{\partial V(0, W_0, i)}{\partial W_0} = p_0^{(i)} = u_c(c_i(0), q_i(0))$. Since the Hamiltonian is concave in $c_i(t)$ and $W_i(t)$, the necessary conditions for optimality are also sufficient (Seierstad and Sydsaeter, 1977).

To analyze the value of health and longevity, we follow Rosen (1988). Let $\delta_{ij}(t)$ be a perturbation on the transition rate, $\lambda_{ij}(t)$, $0 \leq t \leq T$, where $\sum_{j>i}\int_0^T \delta_{ij}(t)dt = 1$. The impact of a small (ε) perturbation on the likelihood of exiting state i is:

$$\tilde{S}^{\varepsilon}(i,t) = \exp\left[-\int_0^t \sum_{j>i}\left(\lambda_{ij}(s) - \varepsilon\delta_{ij}(s)\right)ds\right], \text{ where } \varepsilon > 0 \tag{6}$$

The marginal value of preventing illness or death is equal to $\left.\frac{\partial V/\partial\varepsilon}{\partial V/\partial W}\right|_{\varepsilon=0}$, the marginal rate of substitution between longer life and wealth. The next two lemmas provide the two components of this marginal value expression.

Lemma 2. *The marginal utility of preventing illness or death in state i is given by:*

$$\left.\frac{\partial V(0,W_0,i)}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T e^{-\rho t}\tilde{S}(i,t)\Bigg[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)\,ds\right)\left(u(c_i(t),q_i(t))+\sum_{j>i}\lambda_{ij}(t)V(t,W_i(t),j)\right) - \sum_{j>i}\delta_{ij}(t)V(t,W_i(t),j)\Bigg]dt$$

Proof. See Appendix A ∎

Lemma 3. *The marginal utility of wealth in state i is equal to:*

$$\begin{aligned}\frac{\partial V(0,W_0,i)}{\partial W_0} &= u_c(c_i(0),q_i(0)) \\ &= \mathbb{E}\left[e^{(r-\rho)t}\exp\left\{-\int_0^t \mu(s)\,ds\right\} u_c\Big(c(t,W(t),Y_t),q_{Y_t}(t)\Big)\,\Big|\, Y_0=i, W(0)=W_0\right], \forall t>0\end{aligned}$$

Proof. See Appendix A ∎

The first equality in Lemma 3 follows immediately from the first-order condition in state i in the HJB (2). Our proof derives the second equality, which shows that the consumer sets the expected discounted marginal utility of consumption at time t equal to the current marginal utility of wealth. This result is the stochastic analogue of the first-order condition from a conventional (deterministic health risk) model.

Lemma 2 pertains to a marginal reduction in transition rates for all states and times. Consider as a special case perturbing only $\lambda_{i,n+1}(t)$, the mortality rate in state i, and set the perturbation $\delta(\cdot)$ in equation (6) equal to the Dirac delta function, so that the mortality rate is perturbed at $t=0$ and remains unaffected otherwise (Rosen, 1988). This then yields an expression that is commonly known as the value of statistical life (VSL).

Proposition 4. *Set $\delta_{ij}(t)=0\ \forall j<n+1$ in the marginal utility expression given in Lemma 2 and let $\delta_{i,n+1}(t)$ equal the Dirac delta function. Dividing by the marginal utility of wealth given in Lemma 3 yields:*

$$VSL(i) = \mathbb{E}\left[\int_0^T e^{-\rho t}S(t)\frac{u(c(t),q_{Y_t}(t))}{u_c\Big(c(0),q_{Y_0}(0)\Big)}dt\,\Big|\, Y_0=i, W(0)=W_0\right] = \frac{V(0,W_0,i)}{u_c(c_i(0),q_i(0))} \tag{7}$$

Applying the second equality given in Lemma 3 and rearranging yields the following, equiva-

lent expression for VSL in state i:

$$VSL(i) = \int_0^T e^{-rt} v(i,t)\, dt$$

where $v(i,t)$ represents the value of a one-period change in survival from the perspective of current time:

$$v(i,t) = \frac{\mathbb{E}\Big[S(t)\; u(c(t), q_{Y_t}(t))\Big| Y_0 = i, W(0) = W_0\Big]}{\mathbb{E}\Big[S(t)\; u_c\Big(c(t), q_{Y_t}(t)\Big)\Big| Y_0 = i, W(0) = W_0\Big]}$$

Proof. See Appendix A ■

VSL is the value of a marginal reduction in the risk of death in the current period. Put differently, it is the amount that a large group of individuals are collectively willing to pay to eliminate a current risk that is expected to kill one of them. Proposition 4 shows that VSL is proportional to expected lifetime utility, and inversely proportional to the marginal utility of consumption.

We can also value a marginal reduction in the risk of falling ill. As before, it is helpful to choose the Dirac delta function for $\delta(t)$, so that the transition rates are perturbed at $t = 0$ only. Consider a reduction in the transition rate for a single alternative state, $j \leq n+1$, so that $\delta_{ik}(t) = 0\ \forall k \neq j$. Applying these two conditions in Lemma 3 then yields what we term the value of statistical illness, $VSI(i,j)$:

$$VSI(i,j) = \frac{V(0, W_0, i) - V(0, W_0, j)}{u_c(c_i(0), q_i(0))} = VSL(i) - VSL(j)\frac{u_c\Big(c_j(0), q_j(0)\Big)}{u_c(c_i(0), q_i(0))} \tag{8}$$

The interpretation of VSI is analogous to VSL: it is the amount that a large group of individuals are collectively willing to pay in order to eliminate a current disease risk that is expected to befall one of them. Note that if health state j corresponds to death, so that $VSL(j) = VSL(n+1) = 0$, then $VSI(i,j) = VSL(i)$. Thus, VSI is a generalization of VSL.

The values of statistical life and illness depend on how substitutable consumption is at different ages and states. Intuitively, if present consumption is a good substitute for future consumption, then living a longer life is less valuable. Define the elasticity of intertemporal substitution, σ, as:

$$\frac{1}{\sigma} \equiv -\frac{u_{cc}\, c}{u_c}$$

In addition, define the elasticity of quality of life with respect to the marginal utility of consumption as:

$$\eta \equiv \frac{u_{cq}\, q}{u_c}$$

When η is positive, the marginal utility of consumption is higher in healthier states, and vice-versa. Taking logarithms of equation (5), differentiating with respect to t, plugging in the result for the costate equation and its solution, and rearranging yields an expression for the life-cycle profile of consumption:

$$\frac{\dot{c}_i}{c_i} = \sigma(r-\rho) + \sigma\eta\frac{\dot{q}_i}{q_i} - \sigma\lambda_{i,n+1}(t) - \sigma\sum_{j=i+1}^{n}\lambda_{ij}(t)\left[1 - \frac{u_c\left(c(t,W_i(t),j),q_j(t)\right)}{u_c\left(c(t,W_i(t),i),q_i(t)\right)}\right] \tag{9}$$

The first two terms in equation (9) relate the growth rate of consumption to the consumer rate of time preference and to life-cycle changes in the quality of life. The third term shows that consumption growth is a declining function of the current mortality rate, $\lambda_{i,n+1}(t)$. Because the consumer cannot purchase annuities to insure against her uncertain lifetime, higher rates of mortality depress the rate of consumption growth over the life-cycle. Put another way, removing annuity markets "pulls consumption earlier" in the life-cycle (Yaari, 1965). The fourth term in equation (9)—which is absent from the conventional deterministic setting—accounts for the possibility that the consumer might transition to a different health state in the future. The possibility of transitioning to a state with low marginal utility of consumption shifts life-cycle consumption earlier still.

Equation (9) describes consumption dynamics conditional on the individual's health state i. It is not readily apparent from (9) whether modeling health as stochastic causes consumption to shift forward, *on average* across all states, relative to modeling health as deterministic. We confirmed in numerical exercises that modeling health as stochastic has an ambiguous effect on consumption (and VSL), even when holding quality of life constant across states and time.[7]

2.2 The effect of longevity shocks on VSL

This section considers the effect of stochastic changes in expected longevity on VSL. The effect of any accompanying changes in quality of life depends crucially on the relationship between quality of life and the marginal utility of consumption, a phenomenon often referred to as "health state dependence." Because there is no consensus regarding the sign or magnitude of health state dependence, we hold quality of life constant for the time being and return to this issue in Section 2.3.[8] In our quantitative analysis, we conser-

[7] Modeling health as stochastic has a positive effect on lifetime utility because a stochastic environment allows the consumer to adjust consumption after a health shock. Put differently, a deterministic model is equivalent to a stochastic model where the consumer is forced to keep consumption constant across states.

[8] Viscusi and Evans (1990), Sloan et al. (1998), and Finkelstein et al. (2013) find evidence of negative state dependence. Lillard and Weiss (1997) and Edwards (2008) find evidence of positive state dependence.

vatively assume that quality of life increases the marginal utility of consumption, which biases our estimates away from our theoretical predictions.

When quality of life is constant, VSL can increase or decrease following a health state transition, depending on consumer preferences and expectations of future mortality. We isolate the role played by preferences by analyzing a two-state model, where mortality in state 2 is uniformly higher than mortality in state 1. Intuitively, adverse longevity shocks have two countervailing effects on VSL. On the one hand, a shorter lifespan reduces the lifetime utility of life-extension. On the other hand, a shorter lifespan increases current consumption, which lowers marginal utility and thus increases the willingness to pay for health and longevity. The net effect will depend on the curvature of the utility function relative to the curvature of the marginal utility function.

Our first proposition demonstrates that consumption increases when transitioning to a state where current and future mortality are high, holding wealth constant. If wealth falls following the transition, then the proposition may not hold, depending on the size of the wealth shock. However, in our quantitative analysis—where wealth falls after a health shock because of medical spending—we find that non-medical consumption still generally rises. This result is consistent with Smith (1999), who finds that the reduction in wealth following an adverse health shock is larger than the combined effects on out-of-pocket medical spending and lower income.

Proposition 5. *Let there be $n = 2$ states with constant quality of life, so that $q_1(s) = q_2(s) = q\ \forall s$. Assume that the transition rates $\lambda_{12}(s)$ are uniformly bounded (finite), and that the mortality rate is uniformly higher in state 2: $\lambda_{13}(s) < \lambda_{23}(s)\ \forall s$. Suppose the consumer transitions from state 1 to state 2 at time t. Then $c_1(t, W(t), 1) < c_2(t, W(t), 2)$.*

Proof. See Appendix A ■

To analyze the effect of a transition on VSL, we focus on the case where $\dot{c}_i/c_i \leq 0$, so that consumption does not grow for people who stay in the same health state. Prior empirical work suggests this case is a reasonable description for the typical consumer nearing retirement.[9] In our model, constant quality of life and $r \leq \rho$ are sufficient conditions for

Evans and Viscusi (1991) find no evidence of state dependence. Murphy and Topel (2006) assume negative state dependence when performing their empirical exercises, while Hall and Jones (2007) assume state independence.

[9] A typical consumption profile is constrained by low income at early ages, increasing during middle ages when income is high, and then declines during retirement until consumption equals the consumer's pension. This inverted U-shape for the age profile of consumption has been widely documented across different countries and goods (Carroll and Summers, 1991; Banks et al., 1998; Fernandez-Villaverde and Krueger, 2007).

$\dot{c}_i/c_i \leq 0$.[10]

We analyze the effect of a health shock on VSL by comparing a persistently healthy individual to someone who suffers an adverse shock to life expectancy but is otherwise identical. To make headway we must introduce the notion of prudence. The elasticity of intertemporal substitution, σ, measures utility curvature. Prudence, π, is the analogous measure for the curvature of marginal utility (Kimball, 1990):

$$\pi \equiv -\frac{c\, u_{ccc}(\cdot)}{u_{cc}(\cdot)}$$

It will also be convenient to define the elasticity of the flow utility function:

$$\epsilon \equiv \frac{c\, u_c(\cdot)}{u(\cdot)}$$

The utility elasticity, ϵ, is positive when utility is positive. Positive utility ensures well-behaved preferences, and is often enforced by adding a constant to the utility function. Although adding a constant to the utility function does not affect the solution to the consumer's maximization problem, this constant matters for the value of life.[11]

The following proposition provides sufficient conditions for VSL to rise following an adverse shock to longevity.

Proposition 6. *Consider a two-state setting with assumptions set out in Proposition 5. Assume that $r \leq \rho$, and that utility is positive and satisfies the condition:*

$$\pi < \frac{2}{\sigma} + \epsilon \tag{10}$$

Suppose that the consumer transitions from state 1 to state 2 at time t, and that $\lambda_{12}(\tau) = 0\ \forall \tau > t$. Then, $VSL(1,t) < VSL(2,t)$.

Proof. See Appendix A ■

Proposition 6 shows that the effect of longevity shocks on VSL depends on both prudence and the elasticity of intertemporal substitution. Consumers with inelastic demand for current consumption (low σ) prefer to smooth consumption over time because consumption expenditures at different ages are poor substitutes. They therefore have a high

[10]From equation (9), $\frac{\dot{c}_i}{c_i} \leq 0$ when $\lambda_{i,n+1} \geq r - \rho + \eta \frac{\dot{q}_i}{q_i} - \sum_{j=i+1}^{n} \lambda_{ij}(t)\left[1 - \frac{u_c(c(t,W_i(t),j),q_j(t))}{u_c(c(t,W_i(t),i),q_i(t))}\right]$. This condition is satisfied when $r \leq \rho$, quality of life is constant, and the consumer can transition only to states with higher mortality.

[11]Rosen (1988) was the first to point out that the level of utility is an important determinant of the value of life. See also additional discussion on this point in Hall and Jones (2007) and Córdoba and Ripoll (2016).

willingness to pay for life-extension and, all else equal, are more likely to exhibit a rise in VSL following an adverse longevity shock than consumers with more elastic demand. Likewise, consumers with low levels of prudence, π, have marginal utility that decreases rapidly with consumption and produces a high willingness to pay for life-extension following a shock that increases consumption.

Prior studies on the value of life generally assume that 0.5 to 0.8 is a reasonable range for the value of σ (Murphy and Topel, 2006; Hall and Jones, 2007), and recent empirical studies suggest that π is about 2 (Noussair et al., 2013; Christelis et al., 2020). Under these parameterizations, condition (10) will hold whenever utility is positive. Condition (10) is always satisfied by isoelastic utility, provided that utility is positive. That said, the condition is not innocuous: one can easily find linear combinations of isoelastic and polynomial utility functions where VSL declines following an illness.

Thus, VSL can in general rise or fall following an increase in baseline mortality risk. In static models commonly used in prior studies, however, VSL can only rise with baseline risk (Weinstein et al., 1980; Pratt and Zeckhauser, 1996; Hammitt, 2000). This discrepancy arises because these prior studies focus on a one-period setting with two states, alive and dead. In that context, if the marginal utility of consumption is lower in the dead state, then an increase in baseline mortality risk must lower the expected marginal utility of consumption and thus raise the willingness to pay for survival (the "dead-anyway" effect).[12] Proposition 5 confirms that an increase in the risk of death also reduces marginal utility in our dynamic context. However, unlike in the highly simplified static setting, the resulting effect on VSL is ambiguous because of an offsetting decrease in lifetime utility.

2.3 The value of a health unit

Let D_i denote some measure of health for an individual in state i at time 0, such as quality-adjusted life expectancy. We assume this measure is non-negative, equals 0 only when dead, and is independent of consumption, but otherwise impose no restrictions on its form. When VSL rises following a transition from state i to some state j with lower health (e.g., as in Proposition 6), the value per unit of health must rise as well: $VSL(i)/D_i < VSL(j)/D_j$. This section considers a more general case: How does the value of illness risk reduction per unit of health improvement, $VSI(i,j)/(D_i - D_j)$, vary across different baseline health states i and across different potential illness risks j? Unlike in Section 2.2, our analysis here will allow for an arbitrary number of health states and will not require

[12] Let expected utility be equal to $EU = p\,u(0,c) + (1-p)\,u(1,c)$, where $p \in (0,1)$ is the probability of death and the states $\{0,1\}$ represent death and life, respectively. The willingness to pay for a marginal reduction in the probability of dying is given by $VSL = \frac{u(1,c)-u(0,c)}{p\,u_c(0,c)+(1-p)\,u_c(1,c)}$, which increases with p if $u_c(1,c) > u_c(0,c)$.

quality of life to be constant. Instead, our main results will rely on the concavity of the value function.

For simplicity, it is helpful to assume that health states are ordered in terms of severity. Define the optimal value function in (3) to be concave in health states i, j, and k with respect to changes in our health measure D if the following inequality holds:

$$V(0, W_0, j) > D \times V(0, W_0, i) + (1 - D) \times V(0, W_0, k), \text{ where } D = \frac{D_j - D_k}{D_i - D_k}, D_i > D_j > D_k \quad (11)$$

Let states i, j, and k correspond to "healthy," "mildly ill," and "severely ill." The "value function concavity" condition (11) requires that lifetime utility when mildly ill be larger than the weighted average of the lifetime utilities when healthy or severely ill. In other words, the individual is risk averse over illness severity, preferring mild illness with certainty to good health with a risk of severe illness.

Value function concavity will typically be satisfied when differences in the health measures D_i, D_j, and D_k are large. When health differences are small, then (11) may not hold in some special cases. First, if preferences exhibit negative health state dependence ($u_{cq} > 0$), then the utility function $u(\cdot)$ may not be concave over different combinations of quality of life and consumption.[13] Significant changes in quality of life, q, across health states could then cause value function concavity not to hold. Second, value function concavity may not hold if health risks or quality of life profiles follow empirically implausible but theoretically possible paths. For example, if the future risk of cancer is *higher* in a healthy state than a mildly ill state, then (11) may not hold even though current life expectancy in the healthy state is higher than in the mildly ill state. In our quantitative analysis, which employs real-world estimates of health risks and quality of life profiles, we find that value function concavity is satisfied for most elderly health risks, even when preferences exhibit negative state dependence ($u_{cq} > 0$). For analytical examples, readers can consult our supplementary materials, which consider the special case of isoelastic utility.[14] The materials include a proof showing that if state-specific mortality rates and quality of life profiles are constant over time, then value function concavity will always be satisfied provided that transition rates between health states are sufficiently small.

The following proposition states that value function concavity is necessary and sufficient for the prevention of serious illness risk to be worth more per health unit than prevention of mild illness risk. In addition, if the value function is concave and the marginal

[13]Utility is strictly concave over convex combinations of (c, q) when $u_{cc}u_{qq} - 2u_{cq} > 0$.

[14]Those supplementary materials are available at: https://julianreif.com/research/reif.wp.healthrisk.replication.zip. They include an Excel calculator that quantifies the effect of different health risk sizes on value function concavity in a three-state model.

utility of consumption decreases weakly with baseline health severity, then the value of a health unit will rise with baseline health severity.[15]

Proposition 7. *The optimal value function is concave in health states i, j, and k with respect to changes in the health measure D, as described by (11), if and only if the marginal value of reducing illness risk increases with illness severity:*

$$\frac{VSI(i,j)}{D_i - D_j} < \frac{VSI(i,k)}{D_i - D_k} \text{ where } D_i > D_j > D_k$$

In addition, if the value function is concave in health states i, j, and k, and $u_c(c_i(0), q_i(0)) \geq u_c(c_j(0), q_j(0))$, then the value of a marginal health unit is larger in sicker states:

$$\frac{VSI(i,k)}{D_i - D_k} < \frac{VSI(j,k)}{D_j - D_k} \text{ where } D_i > D_j > D_k$$

Proof. See Appendix A ∎

Propositions 6 and 7 both provide conditions under which VSL per unit of health is higher for those in worse health. Proposition 7, however, applies to both VSI and VSL. For example, consider three different ways to improve one's health: a healthy individual quits smoking to reduce her risk of developing lung cancer (ex ante illness risk reduction); a healthy individual reduces her risk of dying by wearing a seat belt (ex ante mortality risk reduction); and a metastatic lung cancer patient reduces her risk of dying by undergoing chemotherapy (ex post mortality risk reduction). Proposition 7 implies that, under value function concavity, the health benefits of smoking cessation are worth less per unit than wearing a seat belt, which in turn is worth less than chemotherapy.

Our results contrast with traditional cost-effectiveness analysis, which assumes that a health unit is equally valuable regardless of baseline health or illness risk severity (Drummond et al., 2015, Chapter 5). But in fact, a constant value arises only when the utility of consumption is constant (Bleichrodt and Quiggin, 1999). In Appendix D, we show that constant utility of consumption arises in the special case where markets are complete, the rate of time preference equals the interest rate, and quality of life is constant.

[15] Proposition 5 provides an example where marginal utility of consumption will be lower for the ill than the healthy. All else equal, this condition is likely to arise when expected survival is lower in the sick state than in the healthy state.

2.4 Incomplete markets

This section extends our analysis to a setting with incomplete insurance markets and life-cycle income fluctuations. Let income, m_{Y_t}, be exogenous and equal to:

$$m_{Y_t} = \delta_{Y_t} - \omega_{Y_t} + \pi_{Y_t}$$

Income in health state Y_t is equal to labor earnings, δ_{Y_t}, minus health care spending, ω_{Y_t}, plus health insurance reimbursements, π_{Y_t}. Borrowing an approach from Reichling and Smetters (2015), we assume the consumer has an option at time zero to purchase a flat lifetime annuity that pays out $\overline{a}_{Y_0} \geq 0$ in all health states and has a price markup of $\xi \geq 0$. The consumer cannot finance the purchase of the annuity using future earnings or sell her annuity after the purchase. Because the market is incomplete, it will not be optimal to fully annuitize except in certain special cases (Davidoff et al., 2005).[16]

The consumer's maximization problem is:

$$V(0, W_0, Y_0) = \max_{c(t), \overline{a}_{Y_0}} \mathbb{E}\left[\int_0^T e^{-\rho t} S(t)\, u(c(t), q_{Y_t}(t))\, dt \,\middle|\, Y_0, W_0 \right]$$

subject to:

$$\begin{aligned} W(0) &= W_0 - (1+\xi)\overline{a}_{Y_0}\, \mathbb{E}\left[\int_0^T e^{-rt} S(t)\, dt \,\middle|\, Y_0 \right], \\ W(t) &\geq 0, \\ \frac{\partial W(t)}{\partial t} &= rW(t) + m_{Y_t}(t) + \overline{a}_{Y_0} - c(t) \end{aligned}$$

The optimal annuity amount is chosen in the consumer's initial state, Y_0, and the net present value of the annuity may change following a transition to a new health state because a fixed payout is worth more to a person with higher life expectancy. We emphasize this relationship in our notation below by writing the value function V as a function of the optimally chosen annuity and remaining wealth. In addition, it is helpful to define the value of a one-dollar annuity at time t in state i as:

$$a(t, i) = \mathbb{E}\left[\int_t^T e^{-r(s-t)} \exp\left\{ - \int_t^s \mu(u)\, du \right\} ds \,\middle|\, Y_t = i \right]$$

[16] Section 3 uses a numerical model to probe the sensitivity of our results to different assumptions about consumer preferences, such as the presence of a bequest motive, which prior studies have argued might also rationalize low observed rates of annuitization.

Incomplete annuity markets and life-cycle income complicate our analysis by introducing the possibility of multiple sets of non-interior solutions within and across states. (See the right panel in Figure A.1 for an example.) For convenience of exposition, we focus on the case where future income is nondecreasing over time and the growth rate of consumption is weakly declining, as illustrated by the left panel in Figure A.1. As discussed in Section 2.2, this case is a reasonable description for the typical consumer nearing retirement. We do not take a stance on the reason underlying the (weakly) negative growth rate in consumption, but we note that it arises in our model under a wide variety of typical parameterizations. Under these conditions, one can derive a simple expression for VSL.

Proposition 8. *Suppose that annuity markets are incomplete as described above, consumption growth is weakly declining ($\frac{\dot{c}_i}{c_i} \leq 0\ \forall i$), and that income, $m_i(t)$, is nondecreasing in t. Then VSL in state i at time 0 is equal to:*

$$VSL(i) = \frac{V(0, W_i(0), \overline{a}_i, i)}{u_c(c_i(0), q_i(0))} - (1+\xi)\overline{a}_i\ a(0,i) \tag{12}$$

Proof. See Appendix A ■

The second term in equation (12)—sometimes referred to as "net savings"—represents the marginal cost to the annuity pool from saving a life and arises because the price of an annuity is linked to survival (Murphy and Topel, 2006). VSL under incomplete markets captures elements of both the uninsured and fully insured cases. When annuities are absent ($\overline{a}_i = 0$), equation (12) simplifies to the uninsured case given by equation (7). Similarly, full annuitization is optimal when $\xi = 0$, $r = \rho$, and quality of life and future income are constant, in which case equation (12) simplifies to the complete markets case given by equation (D.7) in Appendix D.[17]

The following corollary shows that VSI also takes an intermediate form when markets are incomplete.

Corollary 9. *Consider a setting with assumptions set out in Proposition 8. Then the value of a marginal reduction in the risk of transitioning from state i to state j at time 0 is equal to:*

$$\begin{aligned} VSI(i,j) &= \left(\frac{V(0, W_i(0), \overline{a}_i, i) - V(0, W_i(0), \overline{a}_i, j)}{u_c(c_i(0), q_i(0))}\right) - ((1+\xi)\ \overline{a}_i\ a(0,i) - (1+\xi)\ \overline{a}_i\ a(0,j)) \\ &= VSL(i) - \left(\frac{V(0, W_i(0), \overline{a}_i, j)}{u_c(c_i(0), q_i(0))} - (1+\xi)\ \overline{a}_i\ a(0,j)\right) \end{aligned}$$

[17] Remaining wealth at time 0, $W_i(0)$, is zero under full annuitization, which implies $W_0 = (1+\xi)\overline{a}_i\, a(0,i)$.

Proof. See Appendix A ■

The expression for VSI in Corollary 9 is similar to the expression for VSI in the Robinson Crusoe case (see equation 8), except here there is again an extra term that reflects the effect of a change in survival on net savings.

The net savings term in the VSL and VSI expressions presented above arises only because those expressions are evaluated at time $t = 0$, when the annuity is purchased. The term disappears when evaluating VSI and VSL at $t > 0$—or, equivalently, in a setting with life-cycle income but no opportunity to purchase an annuity—because survival changes occurring after the purchase of the annuity do not affect its price.[18] Because the effect of health transitions on the value of life will generally occur at time $t > 0$, we will assume in what follows that life-extension does not affect the annuity's price.[19]

We first consider the special case of full annuitization. Because the marginal utility of consumption is constant across states under full annuitization, an adverse shock to longevity must reduce VSL, as shown by the following proposition.

Proposition 10. *Consider a two-state setting with assumptions set out in Proposition 5. Assume further that $\xi = 0$, $r = \rho$, and that future income and quality of life are constant across both time and states, so that it is optimal for the consumer to fully annuitize. Suppose the consumer transitions from state 1 (healthy) to state 2 (sick) at time t. Then $VSL(1,t) > VSL(2,t)$.*

Proof. See Appendix A ■

From Propositions 6 and 10, it immediately follows that VSL may in general rise or fall following an adverse health shock when markets are incomplete. Unlike in the Robinson Crusoe case, here the direction of the effect can also depend on the degree of annuitization. For example, full annuitization is optimal in our incomplete markets setting when $\xi = 0$, $r = \rho$, and quality of life and future income are constant, in which case Proposition 10 shows that VSL can fall following the health shock. However, when the load, ξ, is sufficiently large then the incomplete markets setting is well-approximated by the Robinson Crusoe case and Proposition 6 will hold, indicating that VSL can rise following the shock.

Finally, we show that our results from Section 2.3 regarding the value of a health unit continue to hold in this incomplete markets setting. Let D_i be a measure of health in state i at time t, where $D_i = 0$ indexes death. Define the value function of a consumer who purchased an optimal annuity in state i to be concave in health states at time t if for

[18] Philipson and Becker (1998) argue that this "moral hazard" effect induces excessive longevity because individuals do not internalize the costs to annuity programs of their increased lifespan.

[19] We derive VSL and VSI for this case in part (i) of the proof of Proposition 8 and Corollary 9.

health states i, j, k with $D_i > D_j > D_k$, the following inequality holds:

$$V(t, W_i(t), \bar{a}_i, j) > D \times V(t, W_i(t), \bar{a}_i, i) + (1 - D) \times V(t, W_i(t), \bar{a}_i, k) \text{ where } D = \frac{D_j - D_k}{D_i - D_k} \quad (13)$$

Corollary 11. *Suppose the optimal value function is concave in health states at time $t > 0$, as described by (13). Then the two conclusions of Proposition 7 hold at time t in a setting with incomplete insurance markets and life-cycle income fluctuations.*

Proof. See Appendix A ∎

2.5 Welfare

This paper studies the willingness to pay for health and longevity, which helps us to understand puzzles such as why individuals invest less in prevention than treatment. Often, however, policymakers must decide how to allocate resources across different people. Who should receive limited supplies of a vaccine against a pandemic? Should a payer with a fixed budget focus resources on the elderly or the young, on the sick or the poor?

In such contexts, economists frequently rely on comparisons of aggregate social surplus, that is, the aggregate sum of willingness to pay. For example, Murphy and Topel (2006) employ this approach in the framework of the standard life-cycle VSL model. Garber and Phelps (1997) rely on it to develop the theory of cost-effectiveness for health interventions. Einav et al. (2010) use it to study the welfare effects of health insurance. Industrial organization economists use it, in the form of deadweight loss comparisons, to evaluate the welfare consequences of market power (Martin, 2019).

While popular among applied economists and policymakers, the aggregate surplus approach has been criticized by welfare theorists for several reasons (Boadway, 1974; Blackorby and Donaldson, 1990). Equity concerns arise because each dollar of surplus is weighted equally, regardless of differences in wealth or income across people; this implicitly places more weight on the utility of wealthier individuals. Aggregation can also produce intransitive rankings of alternative allocations. Heterogeneity in marginal utility across consumers can break the necessary link between growth in aggregate surplus and increases in utility (Martin, 2019). This last point matters little when valuing the prevention of different illnesses, which can be accomplished from the perspective of a single individual, but it does suggest a need for caution when making welfare inferences across individuals residing in different health states.

One alternative solution is to aggregate utilities rather than monetized surplus, but debate persists about how to aggregate in situations involving risk (Fleurbaey, 2010). In a foundational study, Harsanyi (1955) shows that a social welfare function satisfying both rationality and the Pareto principle must be a weighted sum of ex ante individual utilities. However, this utilitarian approach ignores distributional concerns (Diamond, 1967). As a result, one cannot simultaneously satisfy both rationality and the Pareto principle while still pursuing equity. Theorists have argued for abandoning one or the other of these principles. Diamond (1967) advocates minimizing ex ante inequality, but this violates rationality. Adler and Sanchirico (2006) advocate minimizing ex post inequality, but this violates the Pareto principle. In the specific context of VSL, Pratt and Zeckhauser (1996) advocate maximizing ex ante utility, but this ignores equity concerns in light of Diamond's result. We do not aim to resolve this longstanding debate in welfare economics, but instead note that our stochastic model can be incorporated into these different welfare frameworks as desired.

3 Quantitative Analysis

This section quantifies the value of health improvements achieved through prevention or treatment. While our model provides useful insights on its own, some of our theoretical results require either imposing restrictions on the consumer's setting, such as limiting it to two health states, or assuming value function concavity, which cannot be confirmed without data. Therefore, we complement our theory with quantitative analysis calculating the value of health improvements for a consumer with standard preferences and whose mortality, medical spending, and quality of life can vary across 20 different health states.[20] We calculate both VSI and VSL but focus on their normalized values, VSI per QALY and VSL per QALY, which are more easily compared. We develop a model with a closed-form solution, making our analysis more useful to future analysts seeking to quantify VSL in our framework. To that end, all of our data and code are publicly available online.[21]

[20]Our empirical framework is related to a number of papers that study the savings behavior of the elderly (Kotlikoff, 1988; Palumbo, 1999; De Nardi et al., 2010). These prior studies allow health to affect wealth accumulation by including two or three different health states in the model.

[21]They are available at: https://julianreif.com/research/reif.wp.healthrisk.replication.zip.

3.1 Framework

We employ a discrete time analogue of the model from Section 2. There are n health states (excluding death). Denote the transition probabilities between health states by:

$$p_{ij}(t) = \mathbb{P}\left[Y_{t+1} = j | Y_t = i \right]$$

The mortality rate at time t, $d(t)$, depends on the individual's health state:

$$d(t) = \sum_{j=1}^{n} \overline{d}_j(t) \mathbf{1}\{Y_t = j\}$$

where $\left\{\overline{d}_j(t)\right\}$ are given and $\mathbf{1}\{Y_t = j\}$ is an indicator equal to 1 if the individual is in state j at time t and 0 otherwise.[22] The maximum lifespan of a consumer is T, so $d(T) = 1$. We denote the stochastic probability of surviving from time t to time $s \leq T$ as $S_t(s)$, where:

$$\begin{aligned}
&S_t(t) = 1, \\
&S_t(s) = S_t(s-1)(1 - d(s-1)), \; s > t
\end{aligned}$$

Let $c(t)$ and $W(t)$ denote non-medical consumption and wealth in period t, respectively. Quality of life at time t, $q_{Y_t}(t)$, depends on the individual's health state, Y_t. Let ρ denote the rate of time preference, and r the interest rate. We measure health in state i and time t using quality-adjusted life expectancy, defined as:

$$D_i(t) = \mathbb{E}\left[\sum_{j=t}^{T} e^{-\rho(j-t)} q_{Y_j}(j) S_t(j) \middle| Y_t = i \right] \tag{14}$$

We assume annuity markets are absent. This simplification allows us to calculate the value of life using an analytical solution to the consumer's problem. It is possible to incorporate partial annuitization in this setting along the lines discussed in Section 2.4. However, generalization requires numerical optimization, which may necessitate limiting the number of health states included in the model. In our sensitivity analysis, we model the effects of a bequest motive and of decreasing the substitutability of consumption over time, both of which—similar to annuitization—reduce consumption at earlier ages.

[22] Because our mortality data are distinct from our health state transition data, we denote the probability of dying in state i as $\overline{d}_i(t)$ rather than $p_{i,n+1}(t)$, which differs slightly from the notation used in Section 2.

The consumer's maximization problem is:

$$\max_{c(t)} \mathbb{E}\left[\sum_{t=0}^{T} e^{-\rho t} S_0(t)\, u(c(t), q_{Y_t}(t)) \middle| Y_0, W_0\right]$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ W(t+1) &= (W(t) - c(t))\, e^{r(t, Y_t)} \end{aligned}$$

We account for medical spending by allowing the individual's effective interest rate, $r(t, Y_t)$, to depend on her health state, Y_t. Our baseline model sets $r(t, Y_t) = r + \ln[1 - s(t, Y_t)]$, where r is the rate of interest and $s(t, Y_t)$ is the average share of an individual's wealth spent on medical and long-term care in state Y_t at time t.[23] Instead of deducting medical costs from wealth directly, we treat them as modifying the interest rate. This approach allows us to capture the effects of medical spending on wealth, while preserving the closed-form solution that facilitates quantitative analysis. In addition, the approach allows us to model health shocks that reduce lifetime wealth through, for example, job loss. We assume throughout that $r = \rho = 0.03$ (Siegel, 1992; Moore and Viscusi, 1990).

Finally, we assume that utility takes the following isoelastic form:

$$u(c, q) = q\left(\frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma}\right) \tag{15}$$

The quality of life measure has non-negative values of $q \leq 1$, where $q = 1$ indexes perfect health. Utility is positive when non-medical consumption, c, exceeds the subsistence level, $\underline{c}$. Our main specification sets $\gamma = 1.25$ and $\underline{c} = \$5,000$, consistent with the parameterization employed in Murphy and Topel (2006). Under these parameters, the utility function (15) satisfies condition (10) from Proposition 6. As discussed previously, there is no consensus regarding the sign or magnitude of health state dependence, $u_{cq}(\cdot)$. Here, we assume a multiplicative relationship where the marginal utility of non-medical consumption increases with the health-related quality of life (negative state dependence).

[23]We calculate $s(t, Y_t)$ by dividing out-of-pocket spending in health state Y_t at time t by average wealth at time t, as estimated by our model for a healthy individual with no medical spending. Our results are similar if we instead use age-specific wealth estimates from the Health and Retirement Study.

The value function for the consumer's maximization problem is defined as:

$$V(t,w,i) = \max_{c(t)} \mathbb{E}\left[\sum_{s=t}^{T} e^{-\rho(s-t)} S_t(s)\, u(c(s), q_i(s)) \middle| Y_t = i, W(t) = w \right]$$

We reformulate this optimization problem as a recursive Bellman equation:

$$V(t,w,i) = \max_{c(t)} \left[u(c(t), q_i(t)) + \frac{1-\overline{d}_i(t)}{e^{\rho}} \sum_{j=1}^{N} p_{ij}(t)\, V(t+1, (w-c(t))\, e^{r(t,Y_t)}, j) \right]$$

We solve for consumption analytically and then use the formulas derived in Section 2 to calculate the value of life (see Appendix C for details).

There is significant uncertainty among economists regarding the proper values of many of the parameters in our model. The goal of the subsequent analyses is to quantify the economic significance of our insights by applying our model to real-world data using reasonable parameterizations. To investigate the sensitivity of our results to the parameterization of our utility function, we consider specifications with alternative assumptions regarding the elasticity of intertemporal substitution, $1/\gamma$. We also consider an alternative specification that includes a bequest motive. Rather than setting the utility of death to zero, our bequest motive specification follows Fischer (1973) and sets it equal to $u(W(t+1), b(t))$, where $u(\cdot)$ takes the form given in (15), $W(t+1)$ is wealth at death, and the parameter $b(t)$ governs the strength of the bequest motive. We conservatively set $b(t) = 1.2$, the largest value considered in Fischer (1973), for all t.

3.2 Data

We obtain individual-level data on mortality, disease incidence, quality of life, labor earnings, and out-of-pocket medical spending from the Future Elderly Model (FEM), a widely published microsimulation model that combines nationally representative information from the Health and Retirement Study (HRS), the Medical Expenditure Panel Survey (MEPS), the Panel Study of Income Dynamics, and the National Health Interview Survey. The FEM provides a uniquely rich set of information about the US elderly. For instance, while the HRS provides detailed data on health and wealth, it lacks survey questions that would allow us to calculate quality of life using standard survey instruments. To solve this problem, the FEM weaves together validated quality of life estimates from the MEPS and maps them to the HRS using variables common to both databases.

The FEM, which has been released into the public domain, produces estimates for individuals ages 50–100 with different comorbid conditions (see Appendix B). It accounts for six different chronic conditions (cancer, diabetes, heart disease, hypertension, chronic lung disease, and stroke) and six different impaired activities of daily living (bathing, eating, dressing, walking, getting into or out of bed, and using the toilet). We divide the health space within the FEM into $n = 20$ states. Each state corresponds to the number (0, 1, 2, 3 or more) of impaired activities of daily living (ADL) and the number (0, 1, 2, 3, 4 or more) of chronic conditions, for a total of $4 \times 5 = 20$ health states. Health states are ordered first by number of ADLs and then by number of chronic diseases, so that state 1 corresponds to 0 ADLs and 0 chronic conditions, state 2 corresponds to 0 ADLs and 1 chronic condition, and so on (see Columns (1)–(2) of Table 1). This aggregation provides a parsimonious way of incorporating information about functional status and several major diseases.[24]

All ADLs and chronic conditions are permanent, so an individual's transition probabilities are non-zero only for higher-numbered states that have more ADL's and/or more chronic conditions than her current state. Quality of life is measured by the EuroQol five dimensions questionnaire (EQ-5D). These five dimensions are based on five survey questions that elicit the extent of a respondent's problems with mobility, self-care, daily activities, pain, and anxiety/depression. These questions are then combined using weights derived from stated preference data.[25] The result is a single quality of life index, the EQ-5D, which is anchored at 0 (equivalent to death) and 1 (perfect health).

We calculate population-weighted averages by health state and age, and then use those means as inputs for our model. Table 1 provides summary statistics for ages 50 and 70, by health state. At age 50, life expectancy ranges from 30.9 years to 9.1 years, quality of life ranges from 0.88 to 0.54, and average out-of-pocket medical spending ranges from \$686 to \$2,759 per year. Columns (10) and (11) report the probability that an individual exits her health state but remains alive, i.e., acquires at least one new ADL or chronic condition within the following year. The permanence of ADL's and chronic conditions imposes some natural restrictions on state transitions. For example, an individual in state 14 has 2 ADLs and 3 chronic conditions. She can only transition to states 19 and 20, because all others involve fewer ADLs and/or fewer chronic conditions. Health states are relatively persistent, with empirical exit rates never exceeding 15 percent at ages 50

[24] While fully interacting all these variables would provide a more granular state space, it would also result in a very large number of possible states and correspondingly small cell sizes within many of them.

[25] The five dimensions of the EQ-5D are weighted using estimates from Shaw et al. (2005). The specific process for estimating the quality of life score is explained in the FEM technical documentation, which can be found in the supplemental appendix of Agus et al. (2016). The methods used to measure the quality of life are consistent with our assumed utility specification, given in (15).

or 70. State 20 is an absorbing state with an exit rate of 0 percent.

Figure 1 plots average out-of-pocket medical spending for the healthiest and the sickest health states, by age. These data include all inpatient, outpatient, prescription drug, and long-term care spending not paid for by insurance. Spending is higher in sicker health states, and increases greatly at older ages, when long-term care expenses arise (De Nardi et al., 2010). The effect of sickness on out-of-pocket spending is modest in comparison to long-term care costs, causing the overall gap in spending across states to shrink with age.[26]

We estimate our life-cycle model using FEM data for ages 50–100 but focus our discussion below on ages 50–80, where the FEM estimates are more precise and consumption decisions are less affected by our assumption that annuity markets are absent. We assume that the distribution of initial wealth across health states is proportional to labor earnings at age 50. Finally, we calibrate the level of initial wealth by assuming that the population-weighted average VSL at age 50 is $6 million, which matches the value from Murphy and Topel (2006) and is within the range estimated by empirical studies of VSL for working-age individuals (O'Brien, 2018). Our calibration implies that a healthy 50-year-old in state 1 has a VSL of $6.8 million.

3.3 Explaining variation in the value of life

We begin with a simple example. The solid red and dashed blue lines in Figure 2 report VSL and non-medical consumption for a healthy individual who experiences a mild health shock at age 60, suffers a severe health shock at age 70, and then dies at age 75. Each shock produces sudden changes to expected survival, quality of life, and medical spending, as estimated by the FEM. The black X at age 60 reports what this person's VSL would have been absent the first health shock; the second X reports what her VSL at age 70 would have been absent the second shock. The vertical difference between the X and the red VSL line thus represents the effect of the health shock on VSL.

Consumption increases sharply following the two health shocks depicted in Figure 2. There is little change in VSL at age 60. By contrast, VSL rises to $3.1 million following the severe health shock at age 70, which significantly exceeds the counterfactual value of $2.6 million. Overall, this simple example suggests that our results from Propositions 5 and 6—which predict that consumption and VSL will rise following an adverse shock to life expectancy—are relevant in a more general setting where health shocks also reduce

[26]FEM medical spending estimates have been validated by comparing them to estimates from the National Health Expenditure Accounts (see Section 8.2, Appendix B of National Academies of Sciences, Engineering, and Medicine, 2015).

quality of life and wealth.

To characterize the effects of health shocks among the US elderly population more generally, we next conduct a Monte Carlo exercise that begins with 50,000 nationally representative individuals at age 50. Each person's health path then evolves at random according to the nationally representative health transition probabilities estimated by the FEM. At age 50, VSL ranges from \$0.8 million for the small number of individuals in the worst health state to \$6.8 million for those in the best health state. This dispersion results from differences in both initial health and wealth. Traditional theory accounts for variation in wealth, but it is not configured to analyze variation in baseline health states or the effect of future health risks. In order to abstract away from the effects of differences in initial wealth on VSL, the remainder of this section focuses on the 22,214 healthy individuals who were initially in health state 1 at age 50.

Figure 3a illustrates how the distribution of VSL varies over the life cycle for these 22,214 individuals. The figure plots all twenty ventiles and the bottom and top five percentiles of the distribution in light blue, with the mean in bold. Figure 3b provides a fuller picture of the distribution at age 70. There is a long left-tail of sick individuals who, expecting an imminent death, have spent down their wealth. Individuals with above-average VSL are a mix of healthy individuals and newly diagnosed sick individuals who have begun rapidly spending down their wealth. By age 70, the VSL inter-vigintile range spans \$1.9 to \$2.9 million, with the 80th percentile consumer willing to pay 50% more for life-extension than the 20th.

This cohort of 22,214 individuals experiences about 58,000 health shocks between the ages of 50 and 80. Figure 4a displays the distribution of the change in VSL following each of those shocks.[27] On average, a health shock increases VSL by \$130,000 (3.3%). Figure 4b normalizes these results by the individual's quality-adjusted life expectancy. This second plot shows that health shocks increase VSL per QALY by \$88,000 (21%). The distribution is skewed to the right, with the value of a QALY rising by over \$200,000 (58%) in 5 percent of cases. This rise in the willingness to pay for a QALY helps explain why people state a preference for prioritizing the health of the severely ill (Linley and Hughes, 2013).

Figures 4a and 4b analyze the effects of health shocks on the value of life. Most people, however, have not recently experienced a health shock. To characterize variation for the broader population, the dashed blue line in Figure 5 illustrates how VSL at age 70 varies with quality-adjusted life expectancy across our twenty health states.[28] The positive slope

[27] The change is equal to the difference between actual VSL and a counterfactual VSL, where the counterfactual assumes the individual did not experience the health shock. See Figure 2 for a visual example.

[28] Because health states are persistent (see Columns (10)–(11) of Table 1), the averages shown in Figure

indicates that, on average, VSL rises with life expectancy, consistent with recent work finding that VSL is higher for people in better health (Ketcham et al., 2020). However, the solid red line in Figure 5 indicates that VSL per QALY falls rapidly with quality-adjusted life expectancy. Individuals in the worst health state have an average VSL per QALY of \$630,000, over 2.4 times higher than individuals in the healthiest state, where VSL per QALY is \$260,000. These results strongly contradict conventional cost-effectiveness theory—which assumes the value of a QALY is constant—but are consistent with *ad hoc* approaches that increase cost-effectiveness thresholds for treatments of severe diseases.

Finally, we quantify how the value of prevention varies with the severity of illness risk. The dashed blue line in Figure 6 reports VSI's for different illnesses, including death, from the perspective of a healthy 70-year-old. Each value represents the healthy individual's marginal willingness to pay for a reduction in the risk of death or of transitioning to one of the 19 other health states in our model. The values are inversely related to life expectancy in the sick state because it is more valuable in absolute terms to prevent a severe illness than a mild one. A marginal reduction in the risk of transitioning to the worst health state, where quality-adjusted life expectancy is 2.6 QALYs, is worth about \$2.1 million. VSL, which is a special case of VSI where life expectancy is 0 years in the sick state, is \$2.8 million.

The solid red line in Figure 6 reports VSI per QALY. The negative slope indicates that these values increase with the severity of the disease being prevented. Reducing the risk of death (\$260,000 per QALY) is worth 16% more per QALY than reducing the risk of transitioning to health state 2 (\$224,000 per QALY), the mildest possible illness in our model (life expectancy of 9.7 QALYs). Some (short) sections of the red line occasionally have positive slopes, which indicate a violation of value function concavity. Here, these violations may arise because of a negative correlation between life expectancy and quality of life. For example, at age 70 life expectancy in state 9 (5.4 QALYs) is lower than in state 17 (5.6 QALYs), but quality of life in state 9 is higher (see Table 1). Nevertheless, the general concordance between the estimates shown in Figure 6 and the first inequality stated in Proposition 7 provides evidence that value function concavity holds for most elderly health risks when consumer preferences take the form (15). Finally, note that the solid red line in Figure 6 lies below and features a flatter slope than the solid red line in Figure 5. Consumers place less value on QALYs gained via prevention investment, and this value is less sensitive to the severity of the illness prevented.

5 describe individuals who mostly have not experienced a recent health shock. By contrast, Figure 4a described changes in VSL for individuals who had just experienced a shock. For individuals who survive sufficiently long, an adverse shock to longevity must eventually reduce VSL, relative to no shock, because it causes them to spend down their wealth more quickly.

Figure 7 shows how different utility function parameterizations and the presence of a bequest motive affect our estimates. Setting $\gamma = 1.5$, which makes demand for current consumption more inelastic, flattens the life-cycle consumption profile and increases the value of a QALY. Setting $\gamma = 0.8$, by contrast, pulls consumption forward in time and reduces the value of life-extension because consumption at early ages provide a good substitute for consumption at later ages. A bequest motive encourages individuals to delay consumption, because money saved for consumption in old age has the added benefit of increasing bequests in the event of death (Figure 7a). Likewise, it reduces the value of life-extension because death is less costly (Figure 7b).

Figure 8 shows that our results are driven by changes in mortality, not quality of life or medical spending. Setting quality of life equal to 1 (perfect health) and medical spending to 0 for all ages in all health states shifts non-medical consumption to later ages and raises VSL at later ages, but the effect is small when compared to our main estimates (Figures 8a and 8b).

Overall, while these alternative specifications produce meaningful shifts in the absolute values of VSL and VSI, they do not affect our qualitative conclusions. In all cases, the value of a QALY is larger for sicker individuals (Figures 7c and 8c) and rises with the severity of illness risk (Figures 7d and 8d).

4 Conclusion

The economic theory surrounding the value of life has many important applications. Yet, a number of limitations have surfaced over time. The conventional model does not distinguish between prevention and treatment, and fails to explain several empirical facts, such as the apparent preferences of consumers to pay more for life-extension when survival prospects are bleaker.

Our model overcomes these limitations by allowing for multiple health states. Our framework provides a practical tool for policymakers and health agencies, since patient health varies dramatically across diseases and because many health investments involve preventing the deterioration of health rather than reducing an immediate mortality risk. By deriving a closed-form solution, we ease the burden on practitioners, who can readily use our framework to calculate VSL and VSLY in real-world applications. Indeed, the stochastic framework has meaningful real-world implications: Using nationally representative data, we find that the value of a QALY is dramatically higher for people in worse health states, and that an individual is willing to pay more per QALY to prevent more serious health risks.

These findings provide a rational explanation for why many people state preferences for prioritizing the health of the severely ill over other patients and for why it has proven so difficult for policymakers to encourage investments in preventive care (Linley and Hughes, 2013; Reif et al., 2020). Kremer and Snyder (2015) show that heterogeneity in consumer valuations distorts R&D incentives by allowing firms to extract more consumer surplus from treatments than preventives. Our results suggest that differences in private VSL may reinforce this result and further disadvantage incentives to develop preventives. Regarding healthcare resource allocation, they provide support for the notion of a "severity premium" that increases reimbursements for treatments of more severe diseases (Skedgel et al., 2022). They also provide an additional justification for "top-up" insurance policies: the *ex post* willingness to pay for treatment when sick may significantly exceed the *ex ante* willingness to pay for health insurance coverage when healthy, in which case consumers may benefit from a mechanism that permits supplemental payments in the sick state.

Our analysis raises a number of important questions for further research. First, what are our model's implications for the values of health insurance and of medical technology? Technology that improves quality of life can act as insurance by compressing the difference in utility between the sick and healthy states (Lakdawalla et al., 2017). It is less clear how these effects operate in a stochastic life-cycle setting with incomplete markets. Second, what are the most practical strategies for incorporating our insights into the practice of cost-effectiveness? Practitioners have long assumed that a QALY possesses a constant value. While flawed, this approach is simpler to implement than allowing the value to depend on health histories and illness severity. Translational research should focus on practical strategies for aligning cost-effectiveness practice with the generalized theory of the value of life. Finally, what are the implications for the empirical literature on VSL? Prior studies have assumed that health histories can be ignored when estimating VSL (Hirth et al., 2000; Mrozek and Taylor, 2002; Viscusi and Aldy, 2003), but more recent research suggests otherwise (Ketcham et al., 2020). This missing insight may be one reason for the widely disparate empirical estimates of VSL.

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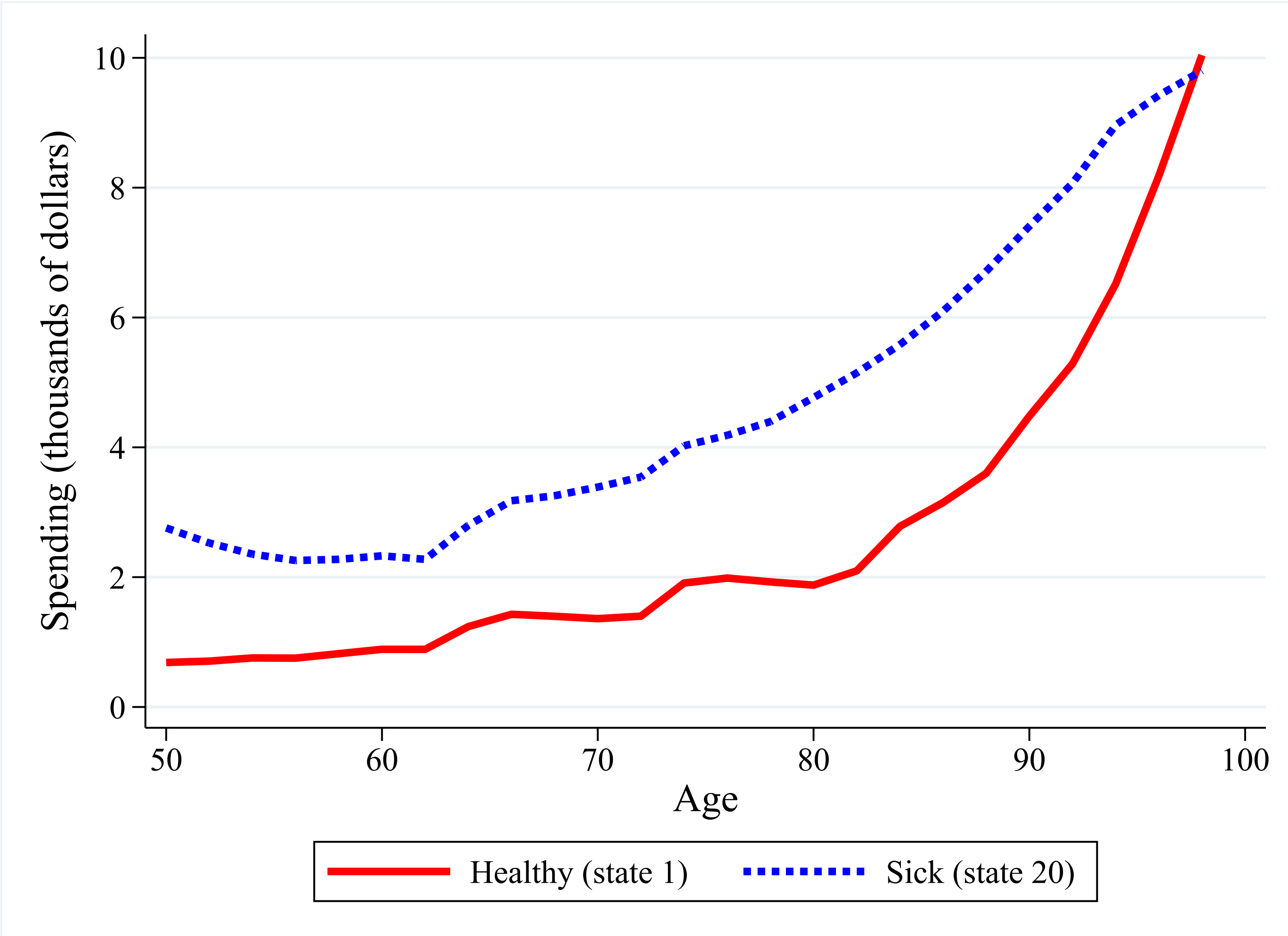
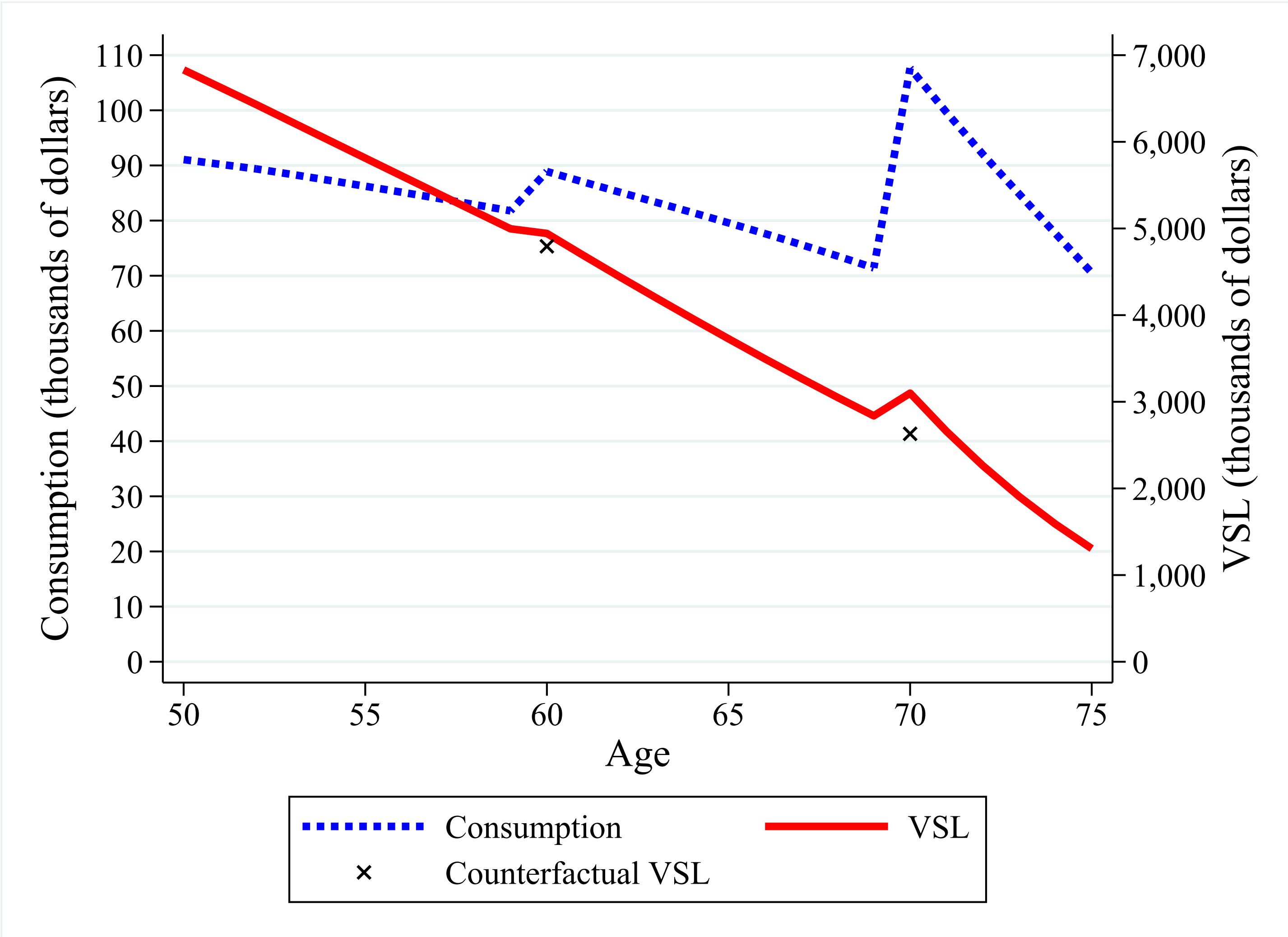
Figure 1: Average annual out-of-pocket medical spending, by age



Notes: These medical spending estimates are produced by the Future Elderly Model (FEM) and include all inpatient, outpatient, prescription drug, and long-term care spending not paid for by insurance. Health state 1 describes healthy individuals with no impaired activities of daily living (ADL) and no chronic conditions. Health state 20 describes very ill individuals with three or more impaired ADLs and four or more chronic conditions. Additional characteristics for these health states are provided in Table 1.

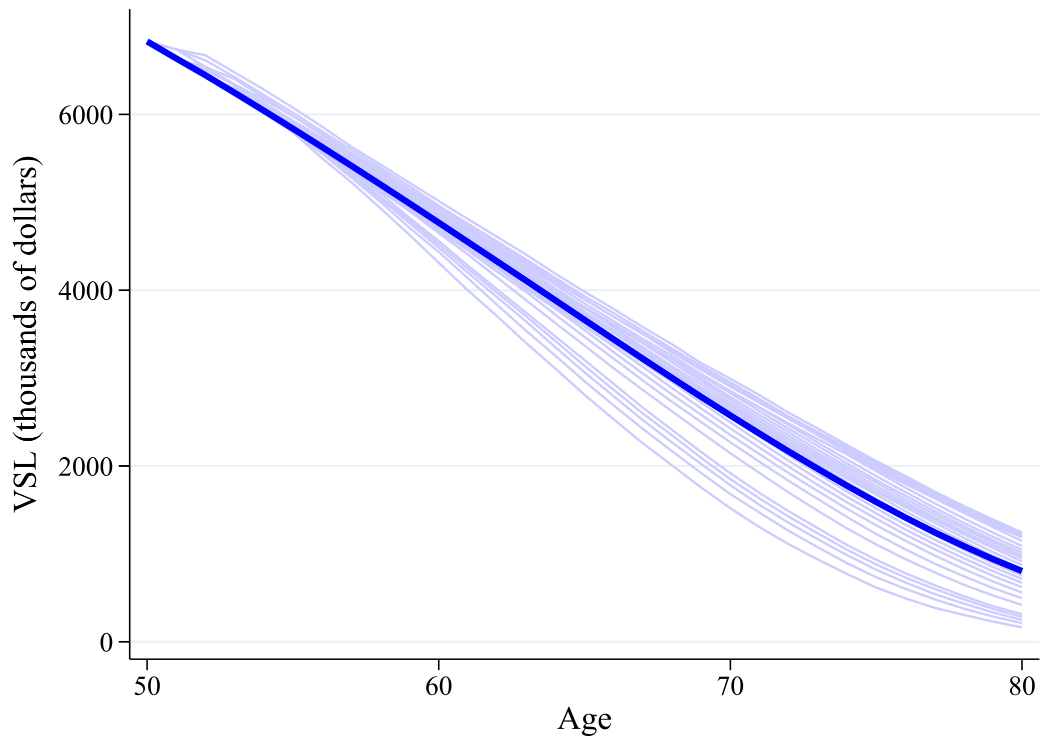
Figure 2: Consumption and the value of statistical life for an individual who suffers two health shocks



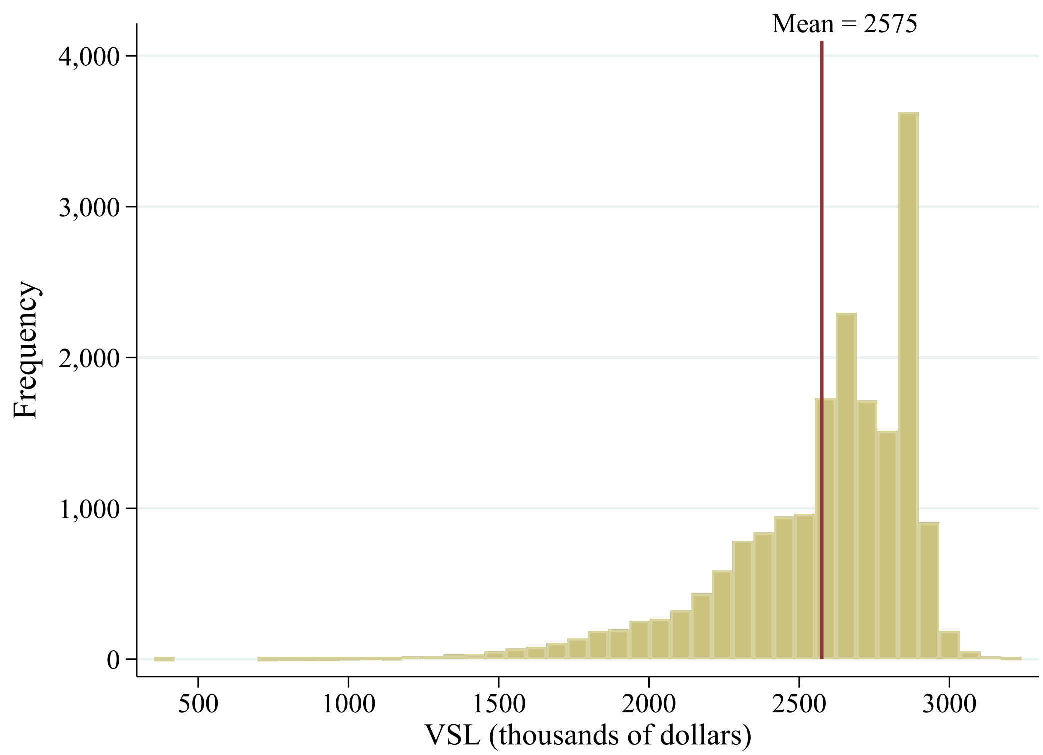
Notes: This figure plots an individual's non-medical consumption (dashed blue line) and value of statistical life (VSL, solid red line) in a setting where mortality, quality of life, and medical spending are stochastic. The individual is healthy at age 50, but then falls ill twice, once at age 60 and then again at age 70. At age 60, the illness impairs one activity of daily living (ADL). At age 70, she is diagnosed with three chronic conditions and one additional impaired ADL. Equivalently, she transitions from state 1 to state 6 at age 60, and then from state 6 to state 14 at age 70 (see Table 1). The individual dies at age 75. The two black X's report VSL for the counterfactual where the individual did not suffer a health shock at age 60 or age 70, respectively. The vertical difference between the black X and the solid red VSL line equals the effect of the health shock on VSL.

Figure 3: Health risk produces significant heterogeneity in the value of statistical life

(a) VSL over time among initially identical healthy adults



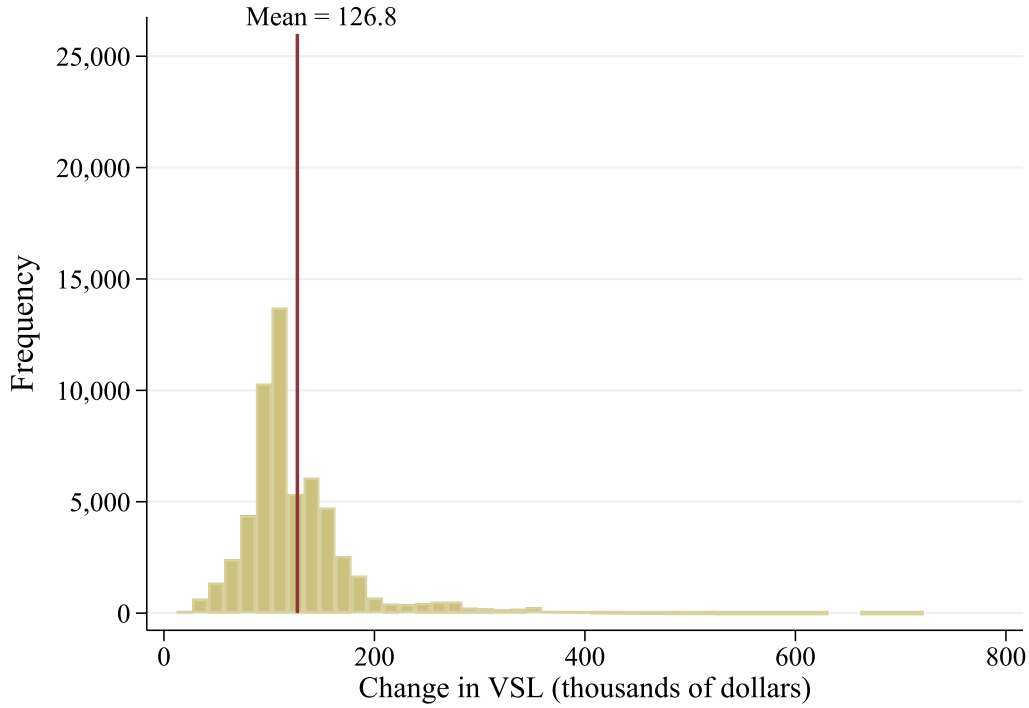
(b) VSL at age 70 among these initially identical adults



Notes: This figure reports VSL statistics for 22,214 initially identical individuals who at age 50 were all healthy and had the same wealth. These individuals then randomly suffer health shocks as they age. Panel (a) plots the twenty ventiles and the bottom and top five percentiles of VSL for this population in light blue, and the mean in dark blue. Panel (b) plots the VSL distribution at age 70. Probabilities and characteristics of the health shocks are estimated by the Future Elderly Model (FEM). FEM summary statistics are available in Table 1.

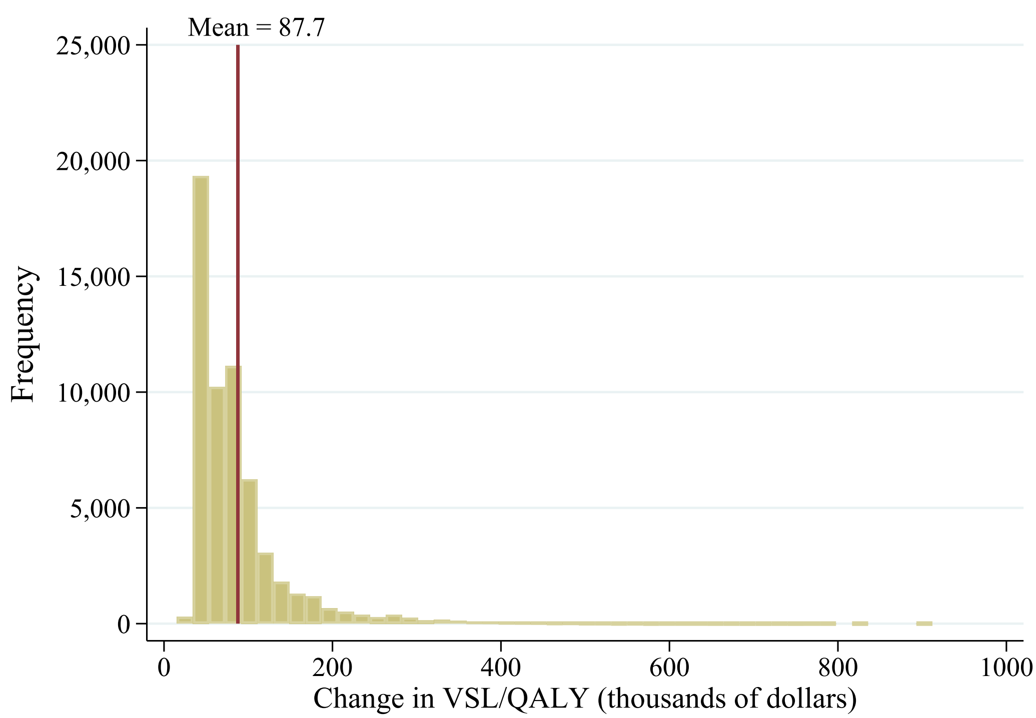
Figure 4: Distribution of changes in VSL following a health shock, ages 50–80

(**a**) Change in VSL



Mean VSL in year prior to shock = 3,832 (thousands of dollars)

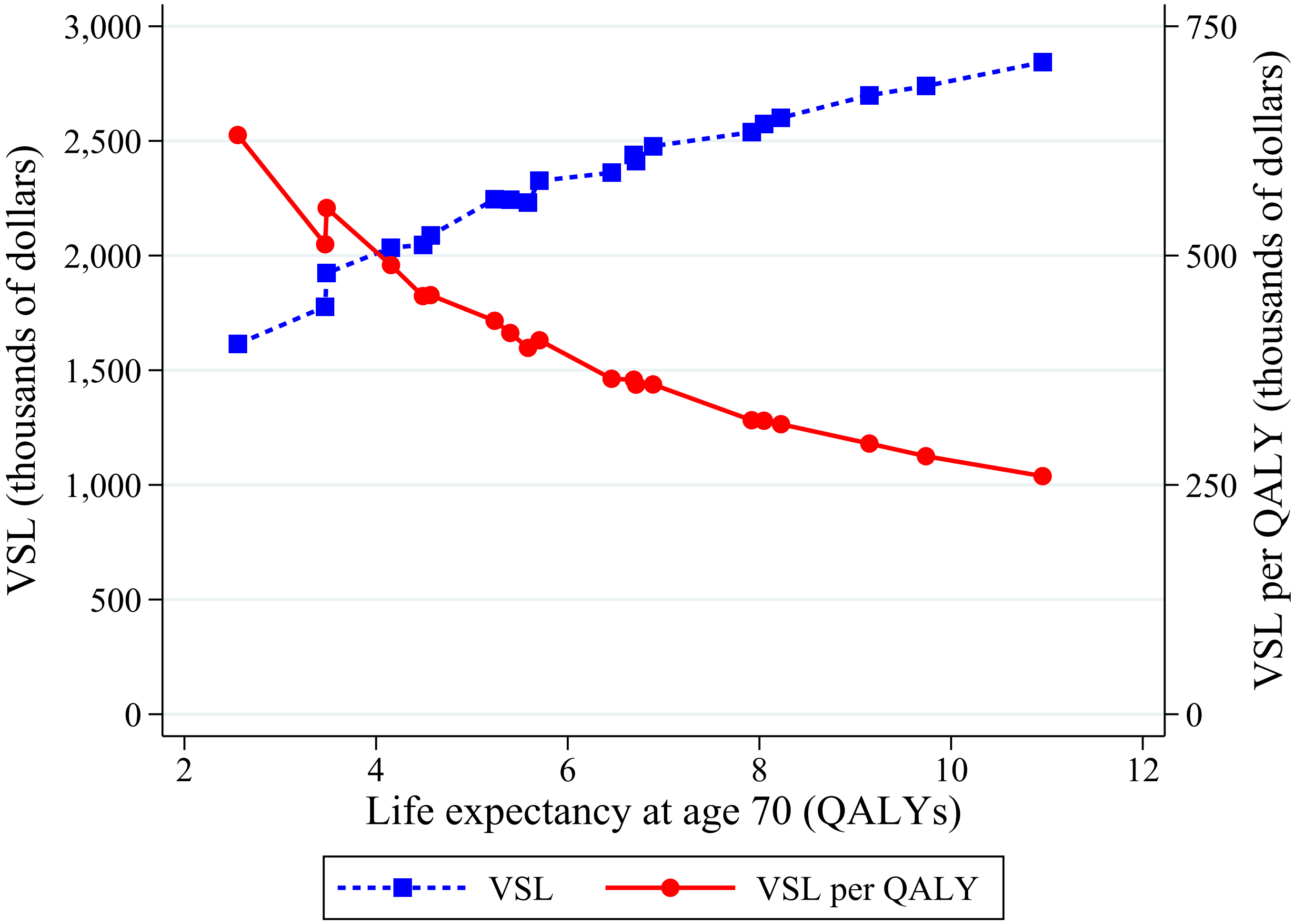
(**b**) Change in VSL per QALY



Mean VSL/QALY in year prior to shock = 348 (thousands of dollars)

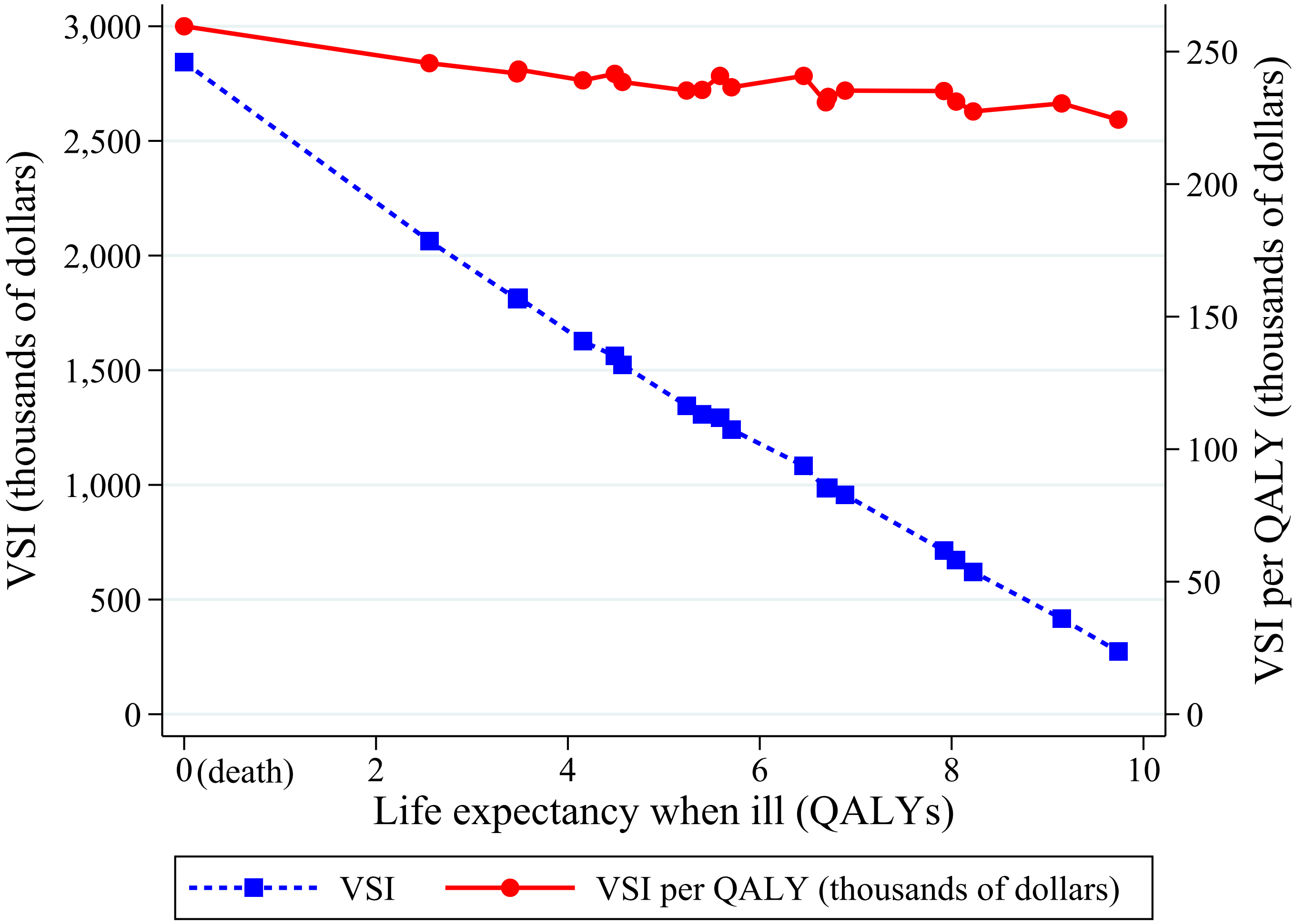
Notes: This figure illustrates the change in VSL following a health shock, relative to a counterfactual with no shock, among the sample of 22,214 initially healthy adults from the Future Elderly Model. These individuals experience 57,981 shocks between the ages of 50–80. Panel (a) plots the distribution of the change in VSL following a health shock. Panel (b) plots the distribution of the change in VSL per quality-adjusted life year (QALY). The vertical red lines report the means of the distributions. QALYs are discounted at a rate of 3 percent. Figure 3a reports how average VSL evolves over the life cycle for this cohort of individuals.

Figure 5: VSL per QALY declines with remaining life expectancy



Notes: This figure presents VSL calculations for a sample of US adults from the Future Elderly Model. The dashed blue line reports average VSL at age 70 for each of the 20 health states described in Table 1. The solid red line normalizes that value by the life expectancy for a person in that health state. Life expectancy is measured in units of quality-adjusted life-years (QALYs) and discounted at a rate of 3 percent. The negative slope of the VSL per QALY line indicates that, on average, sick individuals have a higher willingness to pay for a fixed health gain than healthier individuals.

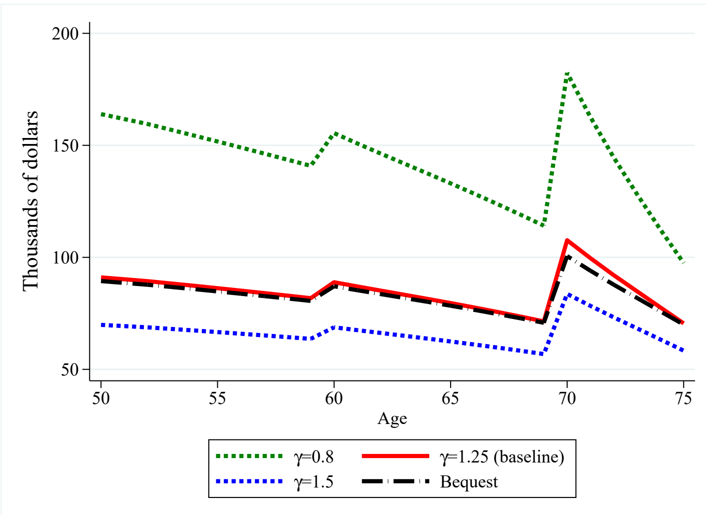
Figure 6: The value of preventing illness at age 70 increases with illness severity



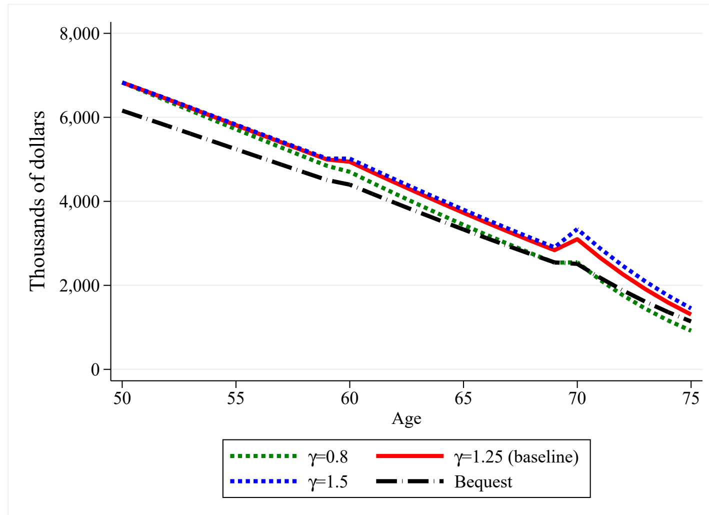
Notes: This figure presents values of statistical illness (VSI) for a sample of US adults from the Future Elderly Model. The dashed blue line reports a healthy (health state 1) 70-year-old's marginal willingness to pay to reduce the risk of different illness, including death (value 0 on the x-axis). The solid red line normalizes that value by the change in life expectancy caused by the illness. Life expectancy is measured in units of quality-adjusted life-years (QALYs) and discounted at a rate of 3 percent. Life expectancy for a 70-year-old in health state 1 is equal to 11.0 QALYs (see Table 1). The negative slope of the VSI per QALY line indicates that individuals are willing to pay more per QALY to reduce severe illness risks than mild illness risks. This result is analogous to how risk-averse individuals are willing to pay more per dollar to insure larger losses.

Figure 7: Sensitivity of results to different parameterizations of utility and to presence of bequest motive

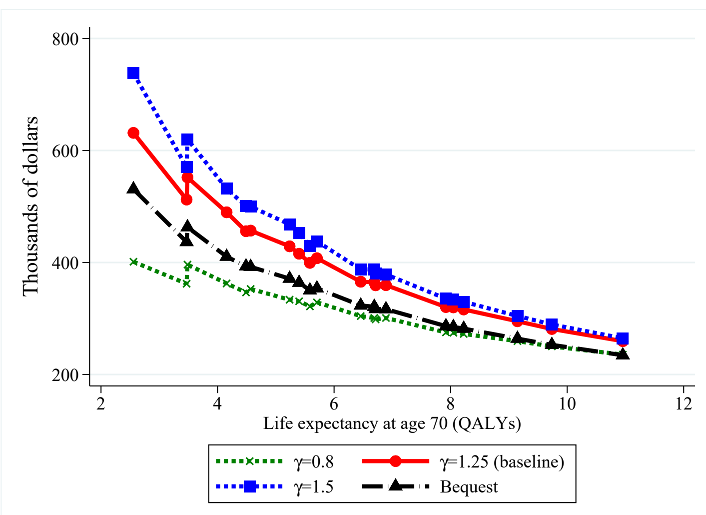
(a) Consumption after two health shocks



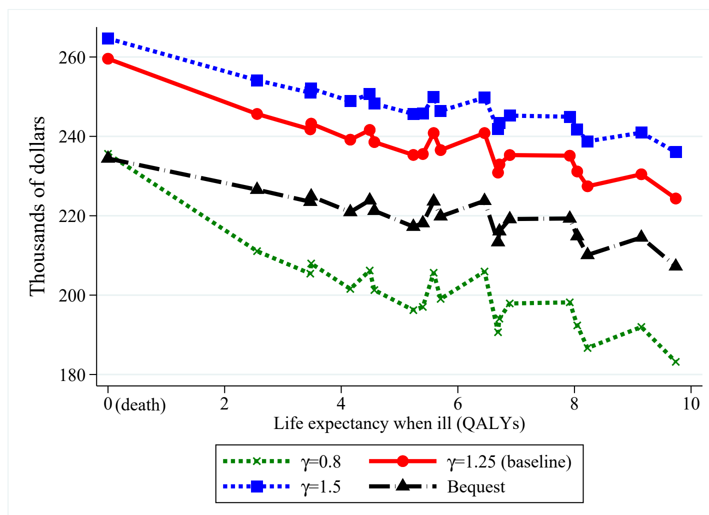
(b) VSL after two health shocks



(c) VSL per QALY declines with remaining life expectancy



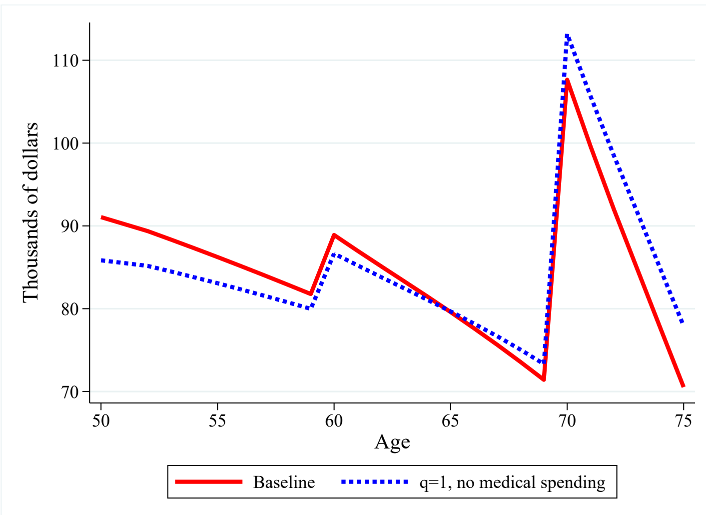
(d) VSI per QALY increases with illness severity



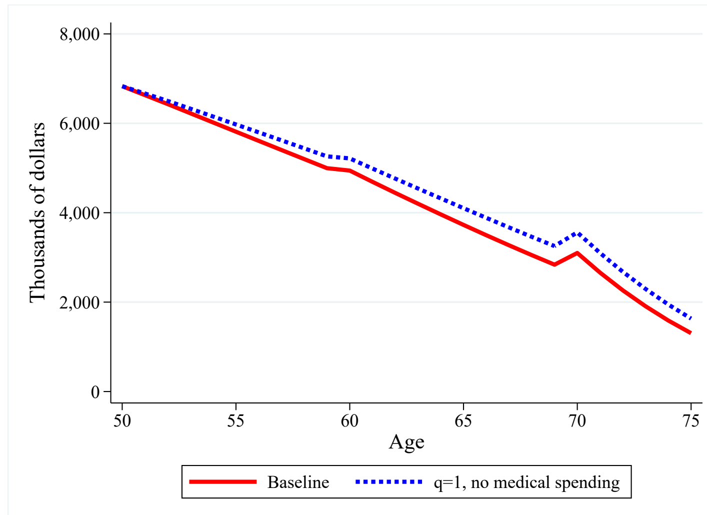
Notes: The solid red lines in panels (a), (b), (c), and (d) replicate the baseline results from Figure 2 (consumption and VSL), Figure 5, and Figure 6, respectively. The dashed green and dashed blue lines present results under the alternative parameter assumptions $\gamma = 0.8$ and $\gamma = 1.5$, respectively, for the utility function (15). The bequest motive specification, depicted by the black dashed line, is based on Fischer (1973) and sets the bequest motive parameter $b(t) = 1.2$ (see Appendix C). Life expectancy is measured in quality-adjusted life-years (QALYs) and discounted at a rate of 3 percent.

Figure 8: Sensitivity of results to quality of life and medical spending

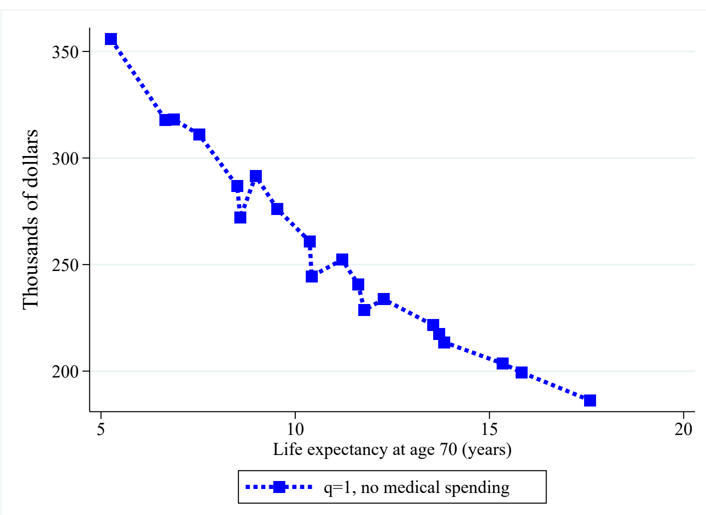
(a) Consumption after two health shocks



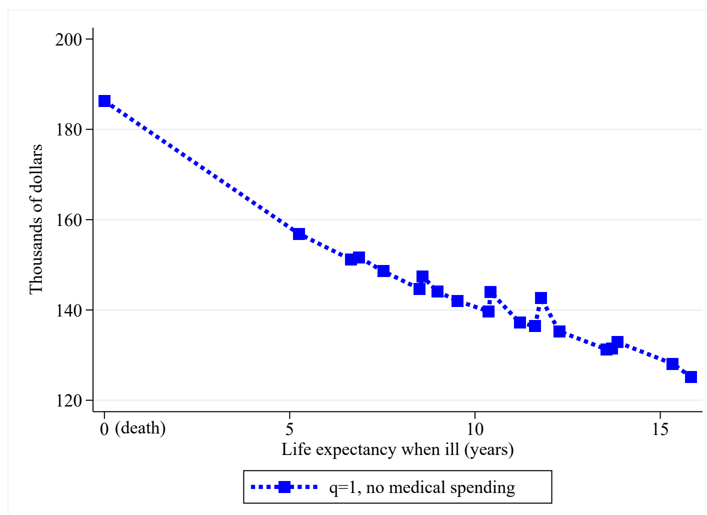
(b) VSL after two health shocks



(c) VSL per life-year declines with remaining life expectancy



(d) VSI per QALY increases with illness severity



Notes: The solid red lines in panels (a) and (b) replicate the baseline results from Figure 2 (consumption and VSL). The dashed blue lines present results when setting quality of life equal to 1 and out-of-pocket medical spending equal to 0 for all ages in all health states, i.e., $q_{Y_t} = q = 1$ and $r(t, Y_t) = r = .03$. (Medical spending is modeled as a modification to the interest rate in our framework.) Life expectancy is measured in life-years and is undiscounted. Panels (c) and (d) omit the baseline results from Figure 5 and Figure 6 because those baseline results were measured in discounted quality-adjusted life-years and thus are not directly comparable to this setting, which sets quality of life equal to 1.

Table 1: Summary means for the Future Elderly Model data, by health state

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		Life expectancy (years)		Life expectancy (QALYs)		Quality of life (EQ-5D)		Medical spending ($)		Exit probability (%)	
Health state	ADLs / CCs	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70
1 (healthy)	0 / 0	30.9	17.6	16.3	11.0	0.88	0.87	686	1,361	4.2	12.6
2	0 / 1	28.2	15.8	14.8	9.7	0.85	0.84	866	1,578	3.6	10.8
3	0 / 2	24.6	13.6	12.8	8.2	0.81	0.80	1,145	1,925	3.6	10.2
4	0 / 3	20.5	11.2	10.7	6.7	0.77	0.76	1,487	2,366	3.9	10.2
5	0 / 4+	16.1	9.0	8.3	5.2	0.73	0.72	2,318	3,193	3.9	7.9
6	1 / 0	26.6	15.3	13.5	9.1	0.83	0.82	598	1,378	6.3	14.7
7	1 / 1	24.0	13.7	12.1	8.0	0.80	0.78	812	1,573	5.7	12.7
8	1 / 2	20.5	11.6	10.2	6.7	0.75	0.75	1,129	1,940	6.1	12.2
9	1 / 3	16.8	9.5	8.3	5.4	0.72	0.71	1,394	2,439	6.4	11.7
10	1 / 4+	13.2	7.5	6.5	4.2	0.67	0.66	2,098	3,287	6.1	8.6
11	2 / 0	24.3	13.8	11.9	7.9	0.78	0.77	585	1,314	7.3	14.3
12	2 / 1	21.5	12.3	10.4	6.9	0.75	0.73	797	1,600	7.5	14.3
13	2 / 2	18.1	10.4	8.7	5.7	0.71	0.69	1,043	1,934	7.5	13.8
14	2 / 3	15.0	8.5	7.1	4.6	0.67	0.66	1,348	2,412	7.5	13.1
15	2 / 4+	11.5	6.7	5.4	3.5	0.63	0.61	1,997	3,322	7.3	10.6
16	3+ / 0	21.9	11.8	10.3	6.5	0.70	0.69	693	1,358	3.4	11.1
17	3+ / 1	19.0	10.4	8.9	5.6	0.66	0.66	948	1,567	2.8	8.5
18	3+ / 2	15.7	8.6	7.3	4.5	0.62	0.62	1,105	1,965	2.3	7.1
19	3+ / 3	12.7	6.9	5.8	3.5	0.58	0.58	1,671	2,472	1.4	5.3
20	3+ / 4+	9.1	5.3	4.1	2.6	0.54	0.54	2,759	3,388	0.0	0.0

Notes: This table reports selected means for the health data obtained from the Future Elderly Model (FEM). Column (1) reports the number of impaired activities of daily living (ADLs) and the number of chronic conditions (CCs), which together define a health state. Columns (2)–(3) report life expectancy in years. Columns (4)–(5) reports life expectancy in QALYs, which is calculated using equation (14) with a 3% discount rate. Columns (6)–(7) report average quality of life as measured by the EQ-5D, where 1 indexes perfect health. Columns (8)–(9) report average annual out-of-pocket medical spending, which includes all inpatient, outpatient, prescription, and long-term care spending not covered by insurance. Columns (10)–(11) report the percentage probability that an individual transitions to a different health state in the following year (excluding death). All impaired ADLs and chronic conditions are permanent, i.e., individuals can transition only to higher-numbered health states. Additional details about the FEM are available in Appendix B.

Online Appendix

"Health Risk and the Value of Life"

Daniel Bauer, University of Wisconsin-Madison
Darius Lakdawalla, University of Southern California and NBER
Julian Reif, University of Illinois and NBER

A Mathematical Proofs

Proof of Lemma 1. Recall that the transition rates $\lambda_{ij}(t) = 0 \ \forall j < i$. The optimization problem in state n is therefore the standard problem with a single health state. We can contemplate a successive solution strategy by starting in state n and then moving sequentially to state $n-1$, $n-2$, etc. Thus, we can consider the deterministic optimization problem for an arbitrary state i by taking $V(t,w,j),\ j > i$, as given (exogenous):

$$V(0,W_0,i) = \max_{c_i(t)} \left\{ \int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right\}$$

subject to:

$$\frac{\partial W_i(t)}{\partial t} = r W_i(t) - c_i(t),$$
$$W_i(0) = W_0$$

Optimal consumption and wealth in state i are denoted by $c_i(t)$ and $W_i(t)$, respectively. Denote the optimal value-to-go function as:

$$\tilde{V}(u, W_i(u), i) = \max_{c_i(t)} \left\{ \int_u^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right\}$$

Setting $\tilde{V}(t, W_i(t), i) = e^{-\rho t} \tilde{S}(i,t) V(t, W_i(t), i)$ then demonstrates that $V(\cdot)$ satisfies the HJB (2) for i. See Theorem 1 and the proof of Theorem 2 in Parpas and Webster (2013) for additional details and intuition behind this result.

■

Proof of Lemma 2. From (3), the marginal utility of preventing an illness or death is:

$$\begin{aligned}
\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon} \int_0^T e^{-\rho t} \exp\left\{ -\int_0^t \sum_{j>i} \left(\lambda_{ij}(s) - \varepsilon \delta_{ij}(s) \right) ds \right\} \left(u\left(c_i^{\varepsilon}(t), q_i(t)\right) + \sum_{j>i} \left(\lambda_{ij}(t) - \varepsilon \delta_{ij}(t) \right) V(t, W_i^{\varepsilon}(t), j) \right) dt \Bigg|_{\varepsilon=0} \\
&= \int_0^T e^{-\rho t} \tilde{S}(i,t) \left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s) ds \right) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) - \sum_{j>i} \delta_{ij}(t) V(t, W_i(t), j) \right] dt \\
&\quad + \int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u_c\left(c_i^{\varepsilon}(t), q_i(t)\right) \frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon} + \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \frac{\partial W_i^{\varepsilon}(t)}{\partial \varepsilon} \right) dt \Bigg|_{\varepsilon=0}
\end{aligned}$$

where $c_i^{\varepsilon}(t)$ and $W_i^{\varepsilon}(t)$ represent the equilibrium variations in $c_i(t)$ and $W_i(t)$ caused by this perturbation.

We conclude the proof by showing that the second term in the last equality is equal to 0. Note that along this path, wealth at time t is equal to:

$$W_i(t) = W_0 e^{rt} - \int_0^t e^{r(t-s)} c_i(s)\, ds$$

which implies $\frac{\partial W_i^{\varepsilon}(t)}{\partial \varepsilon} = -\int_0^t e^{r(t-s)} \frac{\partial c_i^{\varepsilon}(s)}{\partial \varepsilon} ds$. From the solution to the costate equation, we know that:

$$e^{-\rho t} \tilde{S}(i,t) u_c(c_i(t), q_i(t)) = \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i,s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta^{(i)} e^{-rt}$$

Thus, we can rewrite the second term in the expression for $\frac{\partial V}{\partial \varepsilon}\big|_{\varepsilon=0}$ above as:

$$
\begin{aligned}
& \int_0^T \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i,s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds + \theta^{(i)} \right] e^{-rt} \frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon} dt - \int_0^T e^{-\rho t} \tilde{S}(i,t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \int_0^t e^{r(t-s)} \frac{\partial c_i^{\varepsilon}(s)}{\partial \varepsilon} ds\, dt \Bigg|_{\varepsilon=0} \\
= & \int_0^T \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i,s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} \frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon} dt - \int_0^T \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i,s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} \frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon} dt + \int_0^T \theta^{(i)} e^{-rt} \frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon} dt \Bigg|_{\varepsilon=0} \\
= & \ \theta^{(i)} \frac{\partial}{\partial \varepsilon} \underbrace{\int_0^T e^{-rt} c_i^{\varepsilon}(t) dt}_{W_0} \Bigg|_{\varepsilon=0} \\
= & \ 0
\end{aligned}
$$

where the last equality follows from application of the budget constraint.

■

Proof of Lemma 3. We show the result at an arbitrary time t and a future time $\tau > t$:

$$
\frac{\partial V(t, W_i(t), i)}{\partial W_i(t)} = u_c(c_i(t), q_i(t)) = \mathbb{E}\left[e^{(r-\rho)(\tau-t)} \exp\left\{ -\int_t^\tau \mu(s) ds \right\} u_c\left(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau) \right) \Bigg| Y_t = i, W(t) = W_i(t) \right], \forall \tau > t
$$

The proof proceeds by induction on $i \le n$. For the base case $i = n$, in which no state transitions are possible, the solution to the costate equation (4) simplifies to:

$$
\begin{aligned}
p_\tau^{(n)} &= \theta^{(n)} e^{-r\tau} \\
&= \exp\left\{ -\int_0^\tau \rho + \lambda_{n,n+1}(s) ds \right\} u_c(c_n(\tau), q_n(\tau)) \\
&= \theta^{(n)} e^{-rt} e^{-r(\tau-t)} \\
&= p_t^{(n)} e^{-r(\tau-t)} \\
&= \exp\left\{ -\int_0^t \rho + \lambda_{n,n+1}(s) ds \right\} u_c(c_n(t), q_n(t)) e^{-r(\tau-t)}
\end{aligned}
$$

where the second equality makes use of the first-order condition (5). Using the expressions in the second and the last lines then gives:

$$
u_c(c_n(t), q_n(t)) = e^{r(\tau-t)} e^{-\rho(\tau-t)} \exp\left\{ -\int_t^\tau \lambda_{n,n+1}(s) ds \right\} u_c(c_n(\tau), q_n(\tau))
$$

which shows that the lemma holds for $i = n$.

For the induction step, suppose the lemma is true for $j > i$, $1 \le i \le n-1$. For any subinterval $[0, \tau]$, the solution of the costate equation can be written as:

$$
p_t^{(i)} = \left[\int_t^\tau e^{(r-\rho)s} \exp\left\{ -\int_0^s \sum_{j>i} \lambda_{ij}(u) du \right\} \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta(\tau, i) e^{-rt} \tag{A.1}
$$

where $\theta(\tau, i)$ is a constant that depends on the choice of τ and i. (Take the derivative of $p_t^{(i)}$ with respect to t to verify.) Evaluating equation (A.1) at $t = \tau$ and combining with equation (5) from the main text yields:

$$
p_\tau^{(i)} = \theta(\tau, i) e^{-r\tau} = \exp\left\{ -\int_0^\tau \rho + \sum_{j>i} \lambda_{ij}(s) ds \right\} u_c(c_i(\tau), q_i(\tau))
$$

which implies:

$$\theta(\tau,i) = e^{(r-\rho)\tau} \exp\left\{-\int_0^\tau \sum_{j>i} \lambda_{ij}(s) ds\right\} u_c(c_i(\tau), q_i(\tau)) \tag{A.2}$$

Plugging equations (5) and (A.2) into equation (A.1) yields:

$$\begin{aligned} u_c(c_i(t), q_i(t)) \exp\left\{-\int_0^t \rho + \sum_{j>i} \lambda_{ij}(s) ds\right\} &= \left[\int_t^\tau e^{(r-\rho)s} \exp\left\{-\int_0^s \sum_{j>i} \lambda_{ij}(u) du\right\} \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds\right] e^{-rt} \\ &\quad + e^{-rt} e^{(r-\rho)\tau} \exp\left\{-\int_0^\tau \sum_{j>i} \lambda_{ij}(s) ds\right\} u_c(c_i(\tau), q_i(\tau)) \end{aligned}$$

Since $\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)} = u_c(c(s,W_i(s),j), q_j(s))$ from the first-order condition in the HJB for state j, we obtain:

$$\begin{aligned} u_c(c_i(t), q_i(t)) &= \int_t^\tau e^{(r-\rho)(s-t)} \exp\left\{-\int_t^s \sum_{j>i} \lambda_{ij}(u) du\right\} \sum_{j>i} \lambda_{ij}(s) u_c(c(s, W_i(s), j), q_j(s)) ds + e^{(r-\rho)(\tau-t)} \exp\left\{-\int_t^\tau \sum_{j>i} \lambda_{ij}(s) ds\right\} u_c(c_i(\tau), q_i(\tau)) \\ &= \int_t^\tau e^{(r-\rho)(s-t)} \exp\left\{-\int_t^s \sum_{j>i} \lambda_{ij}(u) du\right\} \sum_{j>i} \lambda_{ij}(s) \mathbb{E}\left[e^{(r-\rho)(\tau-s)} \exp\left\{-\int_s^\tau \mu(s) ds\right\} u_c\left(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)\right) \middle| Y_s = j, W(s) = W_i(s)\right] ds \\ &\quad + e^{(r-\rho)(\tau-t)} \exp\left\{-\int_t^\tau \sum_{j>i} \lambda_{ij}(s) ds\right\} u_c(c_i(\tau), q_i(\tau)) \\ &= \mathbb{E}\left[e^{(r-\rho)(\tau-t)} \exp\left\{-\int_t^\tau \mu(s) ds\right\} u_c\left(c(\tau, W(\tau), Y_\tau), q_{y_\tau}(\tau)\right) \middle| Y_t = i,\ W(t) = W_i(t)\right] \end{aligned}$$

where the second equality follows from the induction hypothesis.

■

Proof of Proposition 4. Choosing the Dirac delta function for $\delta(t)$ in Lemma 2 yields:

$$\begin{aligned} \left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} &= \int_0^T \left[e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right)\right] dt \\ &= \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) u(c(t), q_{Y_t}(t)) dt \middle| Y_0 = i, W(0) = W_0 \right] \end{aligned}$$

Dividing the result by the marginal utility of wealth at time $t = 0$ then yields the expression for VSL given by equation (7):

$$VSL(i) = \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) \frac{u(c(t), q_{Y_t}(t))}{u\left(c(0), q_{Y_0}(0)\right)} dt \middle| Y_0 = i, W(0) = W_0 \right]$$

Applying Lemma 3 for $t = 0$ allows us to rewrite VSL as:

$$\begin{aligned} VSL(i) &= \mathbb{E}\left[\int_0^T e^{-\rho t} \frac{S(t) u(c(t), q_{Y_t}(t))}{\mathbb{E}\left[e^{(r-\rho)t} \exp\left\{-\int_0^t \mu(s) ds\right\} u_c\left(c(t), q_{Y_t}(t)\right) \middle| Y_0 = i, W(0) = W_0\right]} dt \middle| Y_0 = i, W(0) = W_0 \right] \\ &= \mathbb{E}\left[\int_0^T e^{-rt} \frac{S(t) u(c(t), q_{Y_t}(t))}{\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s) ds\right\} u_c\left(c(t), q_{Y_t}(t)\right) \middle| Y_0 = i, W(0) = W_0\right]} dt \middle| Y_0 = i, W(0) = W_0 \right] \end{aligned}$$

Exchanging expectation and integration then yields:

$$VSL(i) = \int_0^T e^{-rt} v(i,t)\,dt$$

where $v(i,t)$ is equal to the expected utility of consumption normalized by the expected marginal utility of consumption:

$$v(i,t) = \frac{\mathbb{E}\Big[S(t)\,u(c(t), q_{Y_t}(t))\Big|\,Y_0 = i, W(0) = W_0\Big]}{\mathbb{E}\Big[S(t)\,u_c\big(c(t), q_{Y_t}(t)\big)\Big|\,Y_0 = i, W(0) = W_0\Big]}$$

■

Proof of Proposition 5. Without loss of generality, we will prove the proposition for the case where the consumer transitions from state 1 to state 2 at time $t = 0$. Because we hold quality of life constant, we omit $q_i(t)$ in the notation below in order to keep the presentation concise.

We want to prove that $c_2(0) \geq c_1(0)$. Assume by way of contradiction that $c_2(0) < c_1(0)$. We will show that this assumption implies $c_2(t) < c_1(t)$ for all $t > 0$, which is a contradiction since the feasible consumption plan $c_1(\cdot)$ dominates $c_2(\cdot)$.

We proceed by inductively constructing a sequence $0 < t_1 < t_2 \ldots$ where for each element in the sequence:

$$\begin{aligned} c_2(t_i) &< c_1(t_i) \\ W_1(t_i) &\leq W_2(t_i) \\ p_{t_i}^{(1)} &< \exp\left\{-\int_0^{t_i} \lambda_{12}(s)ds\right\} p_{t_i}^{(2)} \end{aligned}$$

To construct the sequence, for the base case $i = 1$, we first note that from the first-order condition (5), we obtain:

$$p_0^{(1)} = u_c(c_1(0)) < u_c(c_2(0)) = p_0^{(2)}$$

The costate equation (4) then implies:

$$\begin{aligned} \dot{p}_0^{(1)} &= -p_0^{(1)} r - \lambda_{12}(0) u_c(c_2(0)) \\ &= -p_0^{(1)} \left[r + \lambda_{12}(0) \underbrace{\frac{u_c(c_2(0))}{u_c(c_1(0))}}_{>1} \right] \\ &< -p_0^{(1)} [r + \lambda_{12}(0)] = \left.\frac{\partial g(t)}{\partial t}\right|_{t=0} \end{aligned}$$

where $g(t) = p_0^{(1)} \exp\left\{-\int_0^t r + \lambda_{12}(s)\,ds\right\}$. Hence, there exists a $t_1 > t_0 = 0$ such that:

$$p_t^{(1)} \leq g(t) < p_0^{(2)} \exp\left\{-\int_0^t (r + \lambda_{12}(s))\,ds\right\} = p_t^{(2)} \exp\left\{-\int_0^t \lambda_{12}(s)ds\right\}, \ 0 \leq t \leq t_1$$

which together with the first-order condition (5) implies:

$$e^{-\rho t} \exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{13}(s))\,ds\right\} u_c(c_1(t)) < e^{-\rho t} \exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{23}(s))\,ds\right\} u_c(c_2(t)),\ 0 \leq t \leq t_1$$

so that $c_1(t) > c_2(t)$, $0 \leq t \leq t_1$. This inequality in turn implies $W_1(t_1) \leq W_2(t_1)$.

For the induction step, suppose that the following properties also hold for $i \geq 1$:

$$\begin{aligned} c_2(t_i) &< c_1(t_i) \\ W_1(t_i) &\leq W_2(t_i) \\ p^{(1)}_{t_i} &< \exp\left\{-\int_0^{t_i} \lambda_{12}(s)ds\right\} p^{(2)}_{t_i} \end{aligned}$$

The induction hypothesis implies:

$$c(t_i, W_1(t_i), 2) \leq c(t_i, W_2(t_i), 2) = c_2(t_i) < c_1(t_i)$$

so that:

$$\begin{aligned} \dot{p}^{(1)}_{t_i} &= -p^{(1)}_{t_i} r - e^{-\rho t_i} \tilde{S}(1, t_i)\, \lambda_{12}(t_i)\, u\left(c(t_i, W_1(t_i), 2)\right) \\ &= -p^{(1)}_{t_i} \left[r + \lambda_{12}(t_i) \underbrace{\frac{u_c\left(c(t_i, W_1(t_i), 2)\right)}{u_c\left(c_1(t_i)\right)}}_{>1} \right] \\ &< -p^{(1)}_{t_i} \left[r + \lambda_{12}(t_i) \right] = \left.\frac{\partial \tilde{g}(t)}{\partial t}\right|_{t=0} \end{aligned}$$

with $\tilde{g}(t) = p^{(1)}_{t_i} \exp\left\{-\int_{t_i}^{t} (r + \lambda_{12}(s))\, ds\right\}$. Hence, there exists a $t_{i+1} > t_i$ such that:

$$\begin{aligned} p^{(1)}_t &\leq \tilde{g}(t) \\ &< \exp\left\{-\int_0^{t_i} \lambda_{12}(s)\, ds\right\} p^{(2)}_{t_i} \exp\left\{-\int_{t_i}^{t} (r + \lambda_{12}(s))\, ds\right\} \\ &= p^{(2)}_t \exp\left\{-\int_0^{t} \lambda_{12}(s)\, ds\right\}, \; t_i \leq t \leq t_{i+1} \end{aligned}$$

Applying again the first-order condition (5) for all $t_i \leq t \leq t_{i+1}$ yields:

$$\exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{13}(s))\, ds\right\} u_c(c_1(t)) < \exp\left\{-\int_0^t (\lambda_{12}(s) + \lambda_{23}(s))\, ds\right\} u_c(c_2(t))$$

which in turn implies $u_c(c_1(t)) < u_c(c_2(t))$ and $c_2(t) < c_1(t)$. Once again, this inequality implies $W_1(t_{i+1}) \leq W_2(t_{i+1})$.

Thus, we have proven the existence of the sequence. We then obtain $c_2(t) < c_1(t)\ \forall t$ by noting that $\{t_i\}_{i \geq 0}$ strictly increases due to the uniformly boundedness condition on $\lambda_{12}(t)$, which is the desired contradiction.

We note that this proof implies that the consumption paths $c_1(t)$ and $c_2(t)$ cross (at most) once. As soon as $c_1(t)$ exceeds $c_2(t)$ for some time t_0, $c_1(t)$ will exceed $c_2(t)$ for $t > t_0$. However, we have that $c_2(t)$ exceeds $c_1(t)$ prior to t_0. In particular, consumption jumps up upon transition at time zero.

■

Proof of Proposition 6. Without loss of generality, consider the case $t = 0$. Applying our assumptions that $r \leq \rho$ and that quality of life is constant to Equation (9) implies that $c_1(t)$ and $c_2(t)$ are decreasing in t. In addition, from Proposition 5 we have that $c_2(0) > c_1(0)$, $c_2(t) > c_1(t)$ for $t \leq t_0$, and $c_2(t) < c_1(t)$ for $t > t_0$, where t_0 was defined in the proof of Proposition 5. Making use of the assumption that no state transitions occur for $t > 0$, we have that:

$$VSL(2,0) = \int_0^T e^{-rt} \frac{S_2(t)\, u(c_2(t))}{S_2(t)\, u_c(c_2(t))} dt = \int_0^T e^{-rt} \frac{u(c_2(t))}{u_c(c_2(t))} dt$$

and:

$$VSL(1,0) = \int_0^T e^{-rt} \frac{u(c_1(t))}{u_c(c_1(t))} dt$$

Let $Y(x) \equiv \frac{u(x)}{u_c(x)}$. Under the stated assumptions on preferences, we have that:

$$Y^{'}(x) = 1 - \frac{u(x)u_{cc}(x)}{(u_c(x))^2} > 0,$$

$$Y''(x) = \frac{2(u_{cc}(x))^2 u(x) - u_c^2(x)u_{cc}(x) - u_c(x)u(x)u_{ccc}(x)}{(u_c(x))^3} > 0$$

Employing Taylor's theorem and making use of the assumption that $c(t)$ is weakly declining in t then implies that for some $\xi(t)$ that lies in-between $c_1(t)$ and $c_2(t)$:

$$\begin{aligned}
VSL(2,0) &= \int_0^T e^{-rt}\, Y(c_2(t))\, dt \\
&= \int_0^T e^{-rt} \left[Y(c_1(t)) + [c_2(t) - c_1(t)]\, Y^{'}(c_1(t)) + \underbrace{\frac{1}{2}[c_2(t) - c_1(t)]^2 Y''(\xi(t))}_{>0} \right] dt \\
&> \int_0^T e^{-rt}\, Y(c_1(t))\, dt + \int_0^{t_0} e^{-rt} Y'(c_1(t))\, \underbrace{[c_2(t) - c_1(t)]}_{\geq 0}\, dt + \int_{t_0}^T e^{-rt} Y'(c_1(t))\, \underbrace{[c_2(t) - c_1(t)]}_{\leq 0}\, dt \\
&\geq \int_0^T e^{-rt} Y(c_1(t))\, dt + \int_0^{t_0} e^{-rt} Y'(c_1(t_0))\, [c_2(t) - c_1(t)]\, dt + \int_0^{t_0} e^{-rt} Y'(c_1(t_0))\, [c_2(t) - c_1(t)]\, dt \\
&= \int_0^T e^{-rt} Y(c_1(t))\, dt + Y'(c_1(t_0)) \underbrace{\left[\int_0^T e^{-rt} c_2(t)\, dt - \int_0^T e^{-rt} c_1(t)\, dt \right]}_{=0} \\
&= \int_0^T e^{-rt} Y(c_1(t))\, dt \\
&= VSL(1,0)
\end{aligned}$$

where the final step follows from the budget constraint.

■

Proof of Proposition 7. The proposition assumes concavity in health states i, j, and k:

$$V(0, W_0, j) > D \times V(0, W_0, i) + (1 - D) \times V(0, W_0, k), \text{ where } D = \frac{D_j - D_k}{D_i - D_k}$$

This condition is equivalent to:

$$\begin{aligned}
V(0, W_0, i) - V(0, W_0, j) \quad &< \quad (1 - D) \times [V(0, W_0, i) - V(0, W_0, k)] \\
\iff \frac{V(0, W_0, i) - V(0, W_0, j)}{D_i - D_j} \quad &< \quad \frac{V(0, W_0, i) - V(0, W_0, k)}{D_i - D_k}
\end{aligned}$$

since $(1 - D) = (D_i - D_j)/(D_i - D_k)$. Dividing both sides of the final expression by $u_c(c_i(0), q_i(0))$ and applying equation (8) yields the first part of the proposition:

$$\frac{VSI(i,j)}{D_i - D_j} < \frac{VSI(i,k)}{D_i - D_k}$$

For the second part, note that concavity in health states i, j, and k implies:

$$\begin{aligned}
V(0, W_0, j) - V(0, W_0, k) &> D \times [V(0, W_0, i) - V(0, W_0, k)] \\
\iff \frac{V(0, W_0, j) - V(0, W_0, k)}{D_j - D_k} &> \frac{V(0, W_0, i) - V(0, W_0, k)}{D_i - D_k} \\
\iff \frac{V(0, W_0, j) - V(0, W_0, k)}{u_c(c_j(0), q_j(0))} \frac{1}{D_j - D_k} &> \frac{u_c(c_i(0), q_i(0))}{u_c(c_j(0), q_j(0))} \frac{V(0, W_0, i) - V(0, W_0, k)}{u_c(c_i(0), q_i(0))} \frac{1}{D_i - D_k} \\
\iff \frac{VSI(j,k)}{D_j - D_k} &> \frac{u_c(c_i(0), q_i(0))}{u_c(c_j(0), q_j(0))} \frac{VSI(i,k)}{D_i - D_k}
\end{aligned}$$

where the second equivalence follows from dividing by $u_c(c_j(0), q_j(0))$. Finally, the assumption that $u_c(c_i(0), q_i(0)) \geq u_c(c_j(0), q_j(0))$ yields the second part of the proposition:

$$VSI(j,k)/(D_j - D_k) > VSI(i,k)/(D_i - D_k)$$

■

Proof of Proposition 8 and Corollary 9. Our goal is to derive expressions for VSL and VSI when annuity markets are incomplete and the consumer is endowed with state-dependent life-cycle income. We first consider in part (i) the case with life-cycle earnings only. This part also provides expressions for the incomplete markets case at time $t > 0$, because after a flat annuity has been purchased it is equivalent to adding a constant to life-cycle earnings. Part (ii) considers the optimal purchase of the annuity and provides expressions for VSL and VSI at time $t = 0$.

(i) No annuity markets

Denote the consumer's earnings in state i at time t as $m_i(t)$. The consumer's maximization problem is again equation (1), but the law of motion for wealth now includes earnings:

$$\begin{aligned}
W(0) &= W_0, \\
W(t) &\geq 0, \\
\frac{\partial W(t)}{\partial t} &= rW(t) + m_{Y_t}(t) - c(t)
\end{aligned}$$

Once again, we solve this stochastic finite-horizon optimization problem by reformulating it as a deterministic optimization problem. Specifically, we consider equation (3), subject to:

$$\begin{aligned}
W_i(0) &= W_0, \\
W_i(t) &\geq 0, \\
\frac{\partial W_i(t)}{\partial t} &= rW_i(t) + m_i(t) - c_i(t)
\end{aligned}$$

The present-value Hamiltonian corresponding to this deterministic problem is:

$$H\left(W_i(t), c_i(t), p_t^{(i)}, \Psi_t^{(i)}\right) = e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) + p_t^{(i)} [rW_i(t) + m_i(t) - c_i(t)] + \Psi_t^{(i)} W_i(t)$$

where $p_t^{(i)}$ is the costate variable for the wealth dynamics in state i and $\Psi_t^{(i)}$ is the multiplier for the wealth constraint. The first-order conditions are:

$$\begin{aligned}
\dot{p}_t^{(i)} &= -\frac{\partial H}{\partial W_i(t)} = -p_t^{(i)} r - e^{-\rho t}\tilde{S}(i,t)\sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} - \Psi_t^{(i)},\\
p_t^{(i)} &= e^{-\rho t}\tilde{S}(i,t) u_c(c_i(t), q_i(t)),\\
\Psi_t^{(i)} &\geq 0,\\
\Psi_t^{(i)} W_i(t) &= 0
\end{aligned}$$

Following Proposition 1 in Leung (1994), one can show the following: the Hamiltonian is regular on $[0, T)$, so optimal consumption $c_i(t)$ is everywhere continuous; the state-variable inequality constraint is of first-order, so $p_t^{(i)}$ is everywhere continuous; and optimal consumption $c_i(t)$ is continuously differentiable when $W_i(t) > 0$ (i.e., when the wealth constraint is not binding).

First, consider the case when $W_i(t) > 0$. Differentiating the first-order condition for consumption with respect to t, plugging in the result for the costate equation and its solution, and then rearranging yields the rate of change in life-cycle consumption. This rate of change, $\frac{\dot{c}_i}{c_i}$, is identical to the one described by equation (9), and is weakly declining by assumption.

The presence of life-cycle earnings introduces the possibility of multiple sets of non-interior solutions (e.g., right panel of Figure A.1). Modeling these scenarios is possible, but cumbersome. As discussed in the main text, we therefore restrict ourselves to considering the case with a single set of non-interior solutions (i.e., a single "kink point", see left panel of Figure A.1). A sufficient (but not necessary) assumption is that consumption growth is weakly declining. We employ that assumption in the following Lemma, which establishes the existence of a single kink point, T_i, where the consumer runs out of wealth.

Lemma A.1. *Assume $m_i(t)$ is non-decreasing. Then there must exist a T_i such that (1) $W_i(t) = 0$ and $c_i(t) = m_i(t)$ for $t \geq T_i$; and (2) $c_i(t) > m_i(t)$ for $t < T_i$. The solution to the costate equation on $[0, T_i]$ is thus:*

$$p_t^{(i)} = \left[\int_t^{T_i} e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s, W_i(s), j)}{\partial W_i(s)}\, ds\right] e^{-rt} + \theta^{(i)} e^{-rt}$$

where $\theta^{(i)} > 0$ is a constant.

Proof. By assumption, $\frac{\dot{c}_i}{c_i} < 0$ whenever $W_i(t) > 0$. Following the same argument as in Proposition 2 of Leung (1994), there is a smallest T_i such that $W_i(t) = 0$ on $[T_i, T]$ and, thus, $c_i(t) = m_i(t)$ on $[T_i, T]$. Since this is the smallest such T_i, there exists an interval $(\underline{T}_i, T_i)$ such that $W_i(t) > 0$ and $c_i(t_0) > m_i(t_0)$ for a t_0 in the vicinity of T_i. Now assume $W_i(\underline{T}_i) = 0$. Then there exists a t_1 in the vicinity of $\underline{T}_i$ such that $c_i(t_1) < m_i(t_1)$. This is a contradiction, since $m_i(t)$ is non-decreasing and $c_i(t)$ is decreasing whenever $W_i(t) > 0$. Hence $W_i(t) > 0$ on $[0, T_i)$ and $c_i(t) > m_i(t)$ for $t \in [0, T_i)$. As in the main text, the solution to the costate equation can be obtained using the variation of the constant method. ■

Because the value of statistical illness (VSI) is a generalization of the value of statistical life (VSL), we again focus on deriving an expression for VSI. Let $\delta_{ij}(t)$ be a perturbation on the transition rate, and consider the impact on survival as described by equation (6). From equation (3), we obtain:

$$\frac{\partial V}{\partial \varepsilon}\bigg|_{\varepsilon=0} = \frac{\partial}{\partial \varepsilon}\left[\int_0^{T_i(\varepsilon)} e^{-\rho t}\tilde{S}^{\varepsilon}(i,t)\left(u\left(c_i^{\varepsilon}(t),q_i(t)\right)+\sum_{j>i}\left(\lambda_{ij}(t)-\varepsilon\delta_{ij}(t)\right)V(t,W_i^{\varepsilon}(t),j)\right)dt+\int_{T_i(\varepsilon)}^{T} e^{-\rho t}\tilde{S}^{\varepsilon}(i,t)\left(u\left(m_i(t),q_i(t)\right)+\sum_{j>i}\left(\lambda_{ij}(t)-\varepsilon\delta_{ij}(t)\right)V(t,0,j)\right)dt\right]\Bigg|_{\varepsilon=0}$$

$$= \int_0^T e^{-\rho t}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)ds\right)\left(u(c_i(t),q_i(t))+\sum_{j>i}\lambda_{ij}(t)V(t,W_i(t),j)\right)-\sum_{j>i}\delta_{ij}(t)V(t,W_i(t),j)\right]dt$$

$$+\underbrace{\int_0^{T_i} e^{-\rho t}\tilde{S}(i,t)\left(u_c(c_i(t),q_i(t))\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\bigg|_{\varepsilon=0}+\sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t,W_i(t),j)}{\partial W_i(t)}\frac{\partial W_i^{\varepsilon}(t)}{\partial \varepsilon}\bigg|_{\varepsilon=0}\right)dt}_{=0}$$

where the second term in the last equality is equal to 0:

$$\int_0^{T_i} e^{-\rho t}\tilde{S}(i,t)\left(u_c(c_i(t),q_i(t))\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\bigg|_{\varepsilon=0}+\sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t,W_i(t),j)}{\partial W_i(t)}\frac{\partial W_i^{\varepsilon}(t)}{\partial \varepsilon}\bigg|_{\varepsilon=0}\right)dt$$

$$= \int_0^{T_i} p_t^{(i)}\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\bigg|_{\varepsilon=0}+e^{-\rho t}\tilde{S}(i,t)\sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t,W_i(t),j)}{\partial W_i(t)}\left[-\int_0^t e^{r(t-s)}\frac{\partial c_i^{\varepsilon}(s)}{\partial \varepsilon}\bigg|_{\varepsilon=0}ds\right]dt$$

$$= \int_0^{T_i}\theta^{(i)}e^{-rt}\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\bigg|_{\varepsilon=0}dt+\int_0^{T_i}\int_t^{T_i} e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}ds e^{-rt}\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\bigg|_{\varepsilon=0}dt$$

$$-\int_0^{T_i}\int_t^{T_i} e^{-\rho s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}ds e^{rs}e^{-rt}\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\bigg|_{\varepsilon=0}dt$$

$$= \theta^{(i)}\frac{\partial}{\partial \varepsilon}\int_0^{T_i} e^{-rt}c_i^{\varepsilon}(t)dt\bigg|_{\varepsilon=0}$$

$$= 0$$

The final equality follows because $W_i(T_i)=0$ (by definition), which in turn implies $0=W_0+\int_0^{T_i}e^{-rt}m_i(t)dt-\int_0^{T_i}e^{-rt}c_i^{\varepsilon}(t)dt$, so that differentiation yields zero. Thus we obtain:

$$\frac{\partial V}{\partial \varepsilon}\bigg|_{\varepsilon=0} = \int_0^T e^{-\rho t}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)ds\right)\left(u(c_i(t),q_i(t))+\sum_{j>i}\lambda_{ij}(t)V(t,W_i(t),j)\right)-\sum_{j>i}\delta_{ij}(t)V(t,W_i(t),j)\right]dt \tag{A.3}$$

Dividing by the marginal utility of wealth yields the value of life-extension. Choosing the Dirac delta function for $\delta_{i,n+1}(t)$ yields VSL, and choosing the Dirac delta function for $\delta_{ij}(t),\ j<n+1$, yields VSI:

$$VSL(i) = \frac{V(0,W(0),i)}{u_c(c_i(0),q_i(0))} \tag{A.4}$$

$$VSI(i,j) = \frac{V(0,W(0),i)-V(0,W(0),j)}{u_c(c_i(0),q_i(0))} \tag{A.5}$$

(ii) Incomplete annuity markets

Now, we introduce a one-time opportunity at time $t=0$ to purchase a flat lifetime annuity at a level $\overline{a}_{Y_0}\geq 0$ with a price markup $\xi\geq 0$. Let $a(t,i)=\mathbb{E}\left[\int_t^T e^{-r(s-t)}\exp\left\{-\int_t^s \mu(u)du\right\}ds\,\middle|\,Y_t=i\right]$ be the expected value of a one-dollar annuity purchased at time t in state i. Note that for any given annuity, $\overline{a}_i$, the consumer's problem can be mapped to the no-annuity case in part (i) above by setting the constraints equal to:

$$W_i(0) = W_0 - (1+\xi)\overline{a}_i \; a(0,i),$$
$$\frac{\partial W_i(t)}{\partial t} = rW_i(t) + m_i(t) + \overline{a}_i - c_i(t)$$

Solving for the optimal fixed annuity then becomes a straightforward static optimization problem:

$$\overline{a}_i^* = \arg\max_{\overline{a}_i} V(0, W_i(0), \overline{a}_i, i)$$

The optimal annuity must satisfy the necessary first-order condition:

$$\frac{\partial V(0, W_i(0), \overline{a}_i, i)}{\partial \overline{a}_i} = \frac{\partial V(0, W_i(0), \overline{a}_i, i)}{\partial W(0)}(1+\xi)a(0,i) \tag{A.6}$$

Because the consumer may favor a non-flat optimal consumption profile, the optimal level of annuitization is likely to be partial even if the markup ξ is equal to zero. However, full annuitization is optimal when $\xi = 0$, $r = \rho$, and quality of life and income are constant.[1]

The value of an annuity depends on a consumer's expected future survival. Life-extension affects the value and cost of a given annuity, and may also affect the level of the optimal annuity. Thus, the effect of the mortality rate perturbation on the marginal utility of life-extension is:

$$\left.\frac{\partial V\left(0, W_i^{\varepsilon}(0), \overline{a}_i^{\varepsilon}, i\right)}{\partial \varepsilon}\right|_{\varepsilon=0} = (\text{A.3}) + \left.\frac{\partial V}{\partial \overline{a}_i}\frac{\partial \overline{a}_i^{\varepsilon}(0)}{\partial \varepsilon}\right|_{\varepsilon=0} + \left.\frac{\partial V}{\partial W_i(0)}\frac{\partial W_i^{\varepsilon}(0)}{\partial \varepsilon}\right|_{\varepsilon=0}$$

where the first term on the right-hand side is equal to equation (A.3) derived in part (i) above for the case with life-cycle earnings but no annuity. Note that:

$$\begin{aligned}
\left.\frac{\partial W_i^{\varepsilon}(0)}{\partial \varepsilon}\right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon}\left(-(1+\xi)\overline{a}_i^{\varepsilon}\int_0^T \tilde{S}^{\varepsilon}(i,t)e^{-rt}\left[1+\sum_{j>i}\Big(\lambda_{ij}(t)-\varepsilon\delta_{ij}(t)\Big)a(t,j)\right]dt\right) \\
&= -(1+\xi)\left.\frac{\partial \overline{a}_i^{\varepsilon}}{\partial \varepsilon}\right|_{\varepsilon=0} a(0,i) - (1+\xi)\overline{a}_i\int_0^T e^{-rt}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)ds\right)\left(1+\sum_{j>i}\lambda_{ij}(t)a(t,j)\right) - \sum_{j>i}\delta_{ij}(t)a(t,j)\right]dt
\end{aligned}$$

Combining this with the first-order condition (A.6) implies that:

$$\left.\frac{\partial V}{\partial \overline{a}_i}\frac{\partial \overline{a}_i^{\varepsilon}(0)}{\partial \varepsilon}\right|_{\varepsilon=0} + \left.\frac{\partial V}{\partial W_i(0)}\frac{\partial W_i^{\varepsilon}(0)}{\partial \varepsilon}\right|_{\varepsilon=0} = -\frac{\partial V}{\partial W_i(0)}(1+\xi)\overline{a}_i\int_0^T e^{-rt}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)\,ds\right)\left(1+\sum_{j>i}\lambda_{ij}(t)a(t,j)\right) - \sum_{j>i}\delta_{ij}(t)a(t,j)\right]dt$$

Thus the marginal utility of life-extension is equal to:

$$\begin{aligned}
\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} = &\int_0^T e^{-\rho t}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)\,ds\right)\left(u(c_i(t),q_i(t)) + \sum_{j>i}\lambda_{ij}(t)V(t,W_i(t),\overline{a}_i,j)\right) - \sum_{j>i}\delta_{ij}(t)V(t,W_i(t),\overline{a}_i,j)\right]dt \\
&- \frac{\partial V}{\partial W_i(0)}(1+\xi)\overline{a}_i\int_0^T e^{-rt}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)\,ds\right)\left(1+\sum_{j>i}\lambda_{ij}(t)a(t,j)\right) - \sum_{j>i}\delta_{ij}(t)a(t,j)\right]dt
\end{aligned}$$

The marginal utility of wealth, $\partial V/\partial W_i(0)$, is equal to $u_c(c_i(0), q_i(0))$ when the solution is interior. Dividing by the marginal

[1] Even in the case of full annuitization, the first-order condition (A.6) holds with strict equality since the consumer is indifferent between an increase in the annuity level or a proportionate increase in baseline wealth.

utility of wealth and rearranging yields the marginal value of life-extension:

$$
\begin{aligned}
&\left.\frac{\partial V/\partial \varepsilon}{\partial V/\partial W}\right|_{\varepsilon=0} \\
&= \int_0^T \tilde{S}(i,t)\left\{\left(\int_0^t \sum_{j>i}\delta_{ij}(s)ds\right)\left[\left(\frac{e^{-\rho t}u(c_i(t),q_i(t))+\sum_{j>i}\lambda_{ij}(t)V(t,W_i(t),\overline{a}_i,j)}{u_c(c_i(0),q_i(0))}\right)-(1+\xi)\,\overline{a}_i\,e^{-rt}\left(1+\sum_{j>i}\lambda_{ij}(t)a(t,j)\right)\right]\right. \\
&\qquad\qquad \left. -\sum_{j>i}\delta_{ij}(t)\left(\frac{V(t,W_i(t),\overline{a}_i,j)}{u_c(c_i(0),q_i(0))}-(1+\xi)\overline{a}_i e^{-rt}a(t,j)\right)\right\}dt
\end{aligned}
$$

Choosing the Dirac delta function for $\delta_{i,n+1}(t)$ yields:

$$
\begin{aligned}
VSL(i) &= \frac{V(0,W_i(0),\overline{a}_i,i)}{u_c(c_i(0),q_i(0))}-(1+\xi)\overline{a}_i\int_0^T \tilde{S}(i,t)e^{-rt}\left(1+\sum_{j>i}\lambda_{ij}(s)a(t,j)\right)dt \\
&= \frac{V(0,W_i(0),\overline{a}_i,i)}{u_c(c_i(0),q_i(0))}-(1+\xi)\overline{a}_i a(0,i)
\end{aligned}
$$

Likewise, choosing the Dirac delta function for $\delta_{ij}(t),\ j<n+1$, yields:

$$
VSI(i,j)=\left(\frac{V(0,W_i(0),\overline{a}_i,i)}{u_c(c_i(0),q_i(0))}-(1+\xi)a(0,i)\overline{a}_i\right)-\left(\frac{V(0,W_i(0),\overline{a}_i,j)}{u_c(c_i(0),q_i(0))}-(1+\xi)a(0,j)\overline{a}_i\right)
$$

■

Proof of Proposition 10. If $\xi=0$, $r=\rho$, and future income and quality of life are constant across both time and states, then it is optimal for the consumer to fully annuitize, in which case optimal consumption will be constant:

$$
c(t)=m_i(t)+\overline{a}_1=\overline{m}_i+\overline{a}_1=\overline{c}
$$

Without loss of generality, consider a transition from state 1 to state 2 at time $t=0+$, the instant after the consumer has purchased her annuity. Hence, we rely on the VSL expression (A.4) from part (i) of the proof of Proposition 8 and Corollary 9. We have:

$$
VSL(1,0)=\mathbb{E}\left[\int_0^T e^{-rt}S(t)\frac{u(\overline{c},q)}{u_c(\overline{c},q)}dt\,\middle|\,Y_0=1\right]=\frac{u(\overline{c},q)}{u_c(\overline{c},q)}a(0,1)
$$

where $a(0,1)$ is the value of a one-dollar annuity at time $t=0$ in state 1 as defined in the main text. Similarly,

$$
VSL(2,0)=\frac{u(\overline{c},q)}{u_c(\overline{c},q)}a(0,2)
$$

By assumption, survival in the healthy state is larger than survival in the sick state: $\mathbb{E}[S(t)|Y_0=1]>\mathbb{E}[S(t)|Y_0=2]$. This assumption implies $a(0,2)<a(0,1)$, which in turn implies $VSL(1,0)>VSL(2,0)$.

■

Proof of Corollary 11. Again, as in the proof of Proposition 10, we consider transitions at time $t=0+$, the instant after the consumer has purchased her annuity. Using the VSI expression (A.5) from part (i) of the proof of Proposition 8 and Corollary 9, we have:

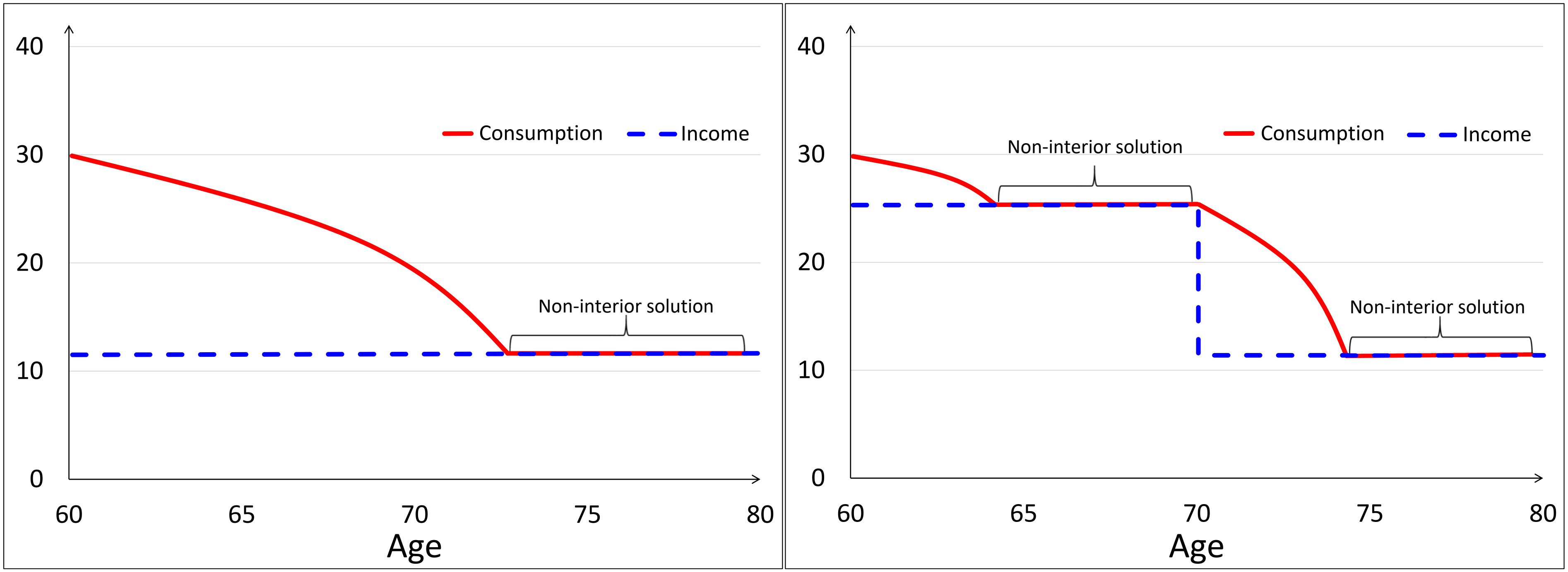
$$
\frac{VSI(i,j)}{D_i-D_j}=\frac{V(0,W_i(0),\overline{a}_i,i)-V(0,W_i(0),\overline{a}_i,j)}{u_c(c_i(0),q_i(0))(D_i-D_j)}.
$$

With condition (13), the results then follow by employing the same arguments as in the proof of Proposition 7. ■

Figure A.1: Illustrative example: survival-contingent income can generate non-interior solutions

(a) One set of non-interior solutions

(b) Two sets of non-interior solutions



Notes: The solution to the consumer's maximization problem may be non-interior in the presence of survival-contingent income. Panel (a) gives an example where there is one set of non-interior solutions. Panel (b) gives an example where there are two sets of non-interior solutions. Income, illustrated by the dashed blue line, includes both labor income and annuity income.

B Future Elderly Model

The empirical exercises presented in Section 3 employ data obtained from the Future Elderly Model (FEM). The FEM is a microsimulation model that projects future health and medical spending for Americans ages 50 and over. It has been used by a variety of researchers and policy analysts to understand the implications of population aging, health trends, new medical technologies, pandemics, and possible health policy interventions in the US, Europe, and Asia (Goldman et al., 2005; Lakdawalla et al., 2005, 2008; Goldman et al., 2009, 2010; Michaud et al., 2011, 2012; Goldman et al., 2013; Goldman and Orszag, 2014; National Academies of Sciences, Engineering, and Medicine, 2015; Chen et al., 2016; Gonzalez-Gonzalez et al., 2017; Leaf et al., 2021; Reif et al., 2021). Detailed technical information about its data sources and methods is available online at:
https://roybalhealthpolicy.usc.edu/fem/technical-specifications/.

The FEM has three core modules. The first is the Replenishing Cohorts module, which predicts economic and health outcomes of new cohorts of 50-year-olds using data from the Panel Study of Income Dynamics, and incorporates trends in disease and other outcomes based on data from external sources, such as the National Health Interview Survey and the American Community Survey. This module generates new cohorts as the simulation proceeds, so that we can measure outcomes for the age 50+ population in any given year.

The second component is the Health Transition module, which uses the longitudinal structure of the Health and Retirement Study (HRS) to calculate transition probabilities across various health states, including chronic conditions, functional status, body-mass index, and mortality. These transition probabilities depend on a battery of predictors: age, sex, education, race, ethnicity, smoking behavior, marital status, employment and health conditions. FEM transitions produce a large set of simulated outcomes, including diabetes, high-blood pressure, heart disease, cancer (except skin cancer), stroke or transient ischemic attack, and lung disease (either or both chronic bronchitis and emphysema), disability, and body-mass index. Disability is measured by limitations in instrumental activities of daily living, activities of daily living, and residence in a nursing home.

Finally, the Policy Outcomes module estimates medical spending, including payments made by insurers (Medicare, Medicaid and Private) and out-of-pocket payments made by individuals. Medical spending for an individual is predicted as a function of health status (chronic conditions and functional status), demographics (age, sex, race, ethnicity and education), nursing home status, and mortality. Estimates are based on spending data from the Medical Expenditure Panel Survey for individuals ages 64 and younger and the Medicare Current Beneficiary Survey for individuals ages 65 and older.

The following example illustrates how the three modules interact. For year 2014, the model begins with the population of Americans ages 50 and over based on nationally representative data from the HRS. Individual-level health and economic outcomes for the next two years are predicted using the Policy Outcomes module. The cohort is then aged two years using the Health Transition Module. Aggregate health and functional status outcomes for those years are then calculated. At that point, a new cohort of 50-year-olds is introduced into the 2016 population using the Replenishing Cohort module, and they join those who survived from 2014 to 2016. This forms the age 50+ population for 2016. The transition model is then applied to this population. The same process is repeated until reaching the last year of the simulation. For our study, we ran the simulation until the year 2064, which gives us complete life-cycle data for ages 50–100 for all people who were ages 50 and over as of 2014.

The projections produced by the FEM have been extensively validated. Mortality forecasts line up closely with published death counts and achieve lower error rates than alternative forecasts used by the Social Security Administration (Leaf et al., 2021). Population, smoking behavior, cancer, diabetes, heart disease, hypertension, lung disease, and stroke forecasts perform well in cross-validation exercises. Medical spending data have been comprehensively tested against national aggregates.

C Supporting Calculations for Quantitative Analysis

This appendix provides the solution to the discrete-time dynamic programming problem described in Section 3.1. This model is solved analytically and provides exact solutions for optimal consumption.

The consumer's problem is:

$$\max_{c(t)} \mathbb{E}\left[\sum_{t=0}^{T} e^{-\rho t} S_0(t)\, u(c(t), q_{Y_t}(t)) + e^{-\rho(t+1)} \left((S_0(t) - S_0(t+1))\, u\left(W(t+1), b(t)\right)\right) \middle| Y_0, W_0\right]$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, \\ W(t+1) &= (W(t) - c(t))\, e^{r(t,Y_t)} \end{aligned}$$

where all variables are defined as in the main text. The strength of the bequest motive is governed by the parameter $b(t)$. We set $b(t) = 0$ in our baseline specification, which assumes no bequest motive (and normalizes utility of death to zero). The utility function is given by equation (15) from the main text:

$$u(c,q) = q\left(\frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma}\right)$$

where $\underline{c}$ is the subsistence level of consumption for a healthy person with no bequest motive. Because optimal consumption is unaffected by affine transformations of utility, we shall initially assume $u(c,q) = q\, c^{1-\gamma}/(1-\gamma)$ when solving the model for consumption.

Define the value function as:

$$V\left(t, W(t), Y_t\right) = \max_{c(s)} \mathbb{E}\left[\sum_{s=t}^{T} e^{-\rho(s-t)} S_t(s)\, u\left(c(s), q_{Y_s}(s)\right) + e^{-\rho(s+1-t)} \left(S_t(s) - S_t\left(s+1\right)\right) u\left(W\left(s+1\right), b(s)\right) \middle| Y_t, W(t)\right]$$

subject to:

$$W\left(s+1\right) = \left(W(s) - c(s)\right) e^{r(s,Y_s)},\ s > t, W(s) \geq 0$$

Then we obtain the following Bellman equation:

$$V(t,w,i) = \max_{c(t)} \left\{ u(c(t), q_i(t)) + e^{-\rho} \overline{d}_i(t)\, u\left((w - c(t))\, e^{r(t,i)}, b(t)\right) + e^{-\rho}\left(1 - \overline{d}_i(t)\right) \sum_{j=1}^{n} p_{ij}(t)\, V\left(t+1, (w - c(t))\, e^{r(t,i)}, j\right) \right\}$$

Proposition C.1. *The value function and the optimal consumption level satisfy:*

$$\begin{aligned} V(t,w,i) &= \frac{w^{1-\gamma}}{1-\gamma} K_{t,i}, \\ c^*(t,w,i) &= w \times c_{t,i} \end{aligned}$$

where:

$$c_{t,i} = \left[1 + e^{-r(t,i)}\left(\frac{e^{r(t,i)}\left[\overline{d}_i(t)\,b(t) + \left(1-\overline{d}_i(t)\right)\left(\sum_{j=1}^{n} p_{ij}(t) K_{t+1,j}\right)\right]}{e^{\rho} q_i(t)}\right)^{\frac{1}{\gamma}}\right]^{-1}, t < T,$$

$$c_{T,i} = \left[1 + e^{-r(t,i)}\left(\frac{e^{r(t,i)}\,b(t)}{e^{\rho}\,q_i(t)}\right)^{\frac{1}{\gamma}}\right]^{-1}$$

and $K_{t,i}$ satisfies the recursion:

$$K_{t,i} = \left[q_i(t)^{\frac{1}{\gamma}} + e^{-r(t,i)}\left[e^{r(t,i)-\rho}\left(\overline{d}_i(t)\,b(t) + \left(1-\overline{d}_i(t)\right)\left(\sum_{j=1}^{n} p_{ij}(t)\,K_{t+1,j}\right)\right)\right]^{\frac{1}{\gamma}}\right]^{\gamma},\ t < T, K_{T,i} = \left[q_i(T)^{\frac{1}{\gamma}} + e^{-r(T,i)}\left(e^{r(T,i)-\rho}b(T)\right)^{\frac{1}{\gamma}}\right]^{\gamma}$$

Proof. See Appendix C.1 ■

When calculating VSL, we incorporate subsistence consumption back into the utility function. In this case, the value function is:

$$\begin{aligned} V(0,w,i) = \sum_{t=0}^{T} e^{-\rho t}\,\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\}\left(q_{Y_t}(t)\,\frac{c(t)^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma}\right)\middle| Y_0 = i, W(0) = w\right] \\ + e^{-\rho(t+1)}\,\mathbb{E}\left[\left(\exp\left\{-\int_0^t \mu(s)ds\right\} - \exp\left\{-\int_0^{t+1}\mu(s)ds\right\}\right)\left(b(t)\,\frac{W(t+1)^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma}\right)\middle| Y_0 = i, W(0) = w\right] \end{aligned} \tag{C.1}$$

Rearranging yields:

$$\begin{aligned} V(0,w,i) &= \sum_{t=0}^{T} e^{-\rho t}\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\} q_{Y_t}(t)\,\frac{c(t)^{1-\gamma}}{1-\gamma}\middle| Y_0 = i, W(0) = w\right] \\ &\quad + e^{-\rho(t+1)}\,b(t)\,\mathbb{E}\left[\left(\exp\left\{-\int_0^t \mu(s)ds\right\} - \exp\left\{-\int_0^{t+1}\mu(s)ds\right\}\right)\frac{W(t+1)^{1-\gamma}}{1-\gamma}\middle| Y_0 = i, W(0) = w\right] \\ &\quad - \frac{\underline{c}^{1-\gamma}}{1-\gamma}\left[q_{Y_0}(0) + e^{-\rho}\,b(0) + \sum_{t=1}^{T} e^{-\rho t}\,\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\}\left(q_{Y_t}(t) + e^{-\rho}b(t) - b(t-1)\right)\middle| Y_0 = i\right]\right] \\ &= \frac{1}{1-\gamma}\left[w^{1-\gamma}K_{0,i} - \underline{c}^{1-\gamma}\left[q_{Y_0}(0) + e^{-\rho}b(0) + \sum_{t=1}^{T} e^{-\rho t}\,\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\}\left(q_{Y_t}(t) + e^{-\rho}b(t) - b(t-1)\right)\middle| Y_0 = i\right]\right]\right] \end{aligned}$$

We can then calculate VSL in state i using the following formula:

$$VSL(i) = \frac{V(0,w,i) - b(0)\left(\frac{w^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma}\right)}{u_c\left(w\,c_{0,i}, q_i(0)\right)} \tag{C.2}$$

The second term in the numerator of (C.2) is the utility at death (the bequest function). When the bequest motive is absent ($b(t) \equiv 0$), the value function simplifies to:

$$V(0,w,i) = \frac{1}{1-\gamma}\left[w^{1-\gamma}K_{0,i} - \underline{c}^{1-\gamma}\underbrace{\sum_{t=0}^{T} e^{-\rho t}\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\} q_{Y_t}(t)\middle| Y_0 = i\right]}_{\text{discounted quality–adjusted life expectancy in state } i}\right]$$

and the expression for VSL simplifies to equation (7) from the main text.

Once one has calculated VSL, it is straightforward to calculate VSI:

Corollary C.2. *The value of a marginal reduction in the probability of transitioning from state i to state j is equal to:*

$$VSI(i,j) = VSL(i) - VSL(j)\frac{q_j(0)\,c_{0,j}^{-\gamma}}{q_i(0)\,c_{0,i}^{-\gamma}} = VSL(i) - \left(\frac{q_j(0)}{q_i(0)}\right)\left(\frac{c_{0,i}}{c_{0,j}}\right)^{\gamma} VSL(j)$$

Proof. See Appendix C.1 ∎

C.1 Proofs

Proof of Proposition C.1. The proof proceeds by induction on $t \le T$. For the base case $t = T$, note that $\overline{d}_i(t) = 1$, so that the first-order condition from the Bellman equation gives:

$$q_i(T)\,c(T)^{-\gamma} = e^{r(T,i)-\rho}\,b(T)(w - c(T))^{-\gamma}\,e^{-r(T,i)\gamma}$$

Rearranging this first-order condition yields:

$$c(T) = \frac{w\,e^{r(T,i)}e^{\frac{(\rho - r(T,i))}{\gamma}}\left(\frac{q_i(T)}{b(T)}\right)^{\frac{1}{\gamma}}}{1 + e^{r(T,i)}e^{\frac{(\rho - r(T,i))}{\gamma}}\left(\frac{q_i(T)}{b(T)}\right)^{\frac{1}{\gamma}}} = w\underbrace{\left[1 + e^{-r(T,i)}\left(\frac{e^{r(T,i)}b(T)}{e^{\rho}q_i(T)}\right)^{\frac{1}{\gamma}}\right]^{-1}}_{c_{T,i}}$$

Hence, we obtain:

$$\begin{aligned}
V(T,w,i) &= \frac{w^{1-\gamma}}{1-\gamma}\left(q_i(T)\,c_{T,i}^{1-\gamma} + e^{-\rho}\,b(T)\,e^{r(T,i)(1-\gamma)}(1 - c_{T,i})^{1-\gamma}\right) \\
&= \frac{e^{-\rho}e^{r(T,i)(1-\gamma)}}{\left[b_T^{\frac{1}{\gamma}} + e^{r(T,i)}e^{\frac{(\rho - r(T,i))}{\gamma}}\,q_i(T)^{\frac{1}{\gamma}}\right]^{-\gamma}} = \left[q_i(T)^{\frac{1}{\gamma}} + e^{-r(T,i)}\left(e^{(r(T,i)-\rho)}b(T)\right)^{\frac{1}{\gamma}}\right]^{\gamma}
\end{aligned}$$

For the induction step, suppose the proposition is true for case $t+1$. We have:

$$V(t,w,i) = \max_c\left\{q_i(t)\frac{c^{1-\gamma}}{1-\gamma} + b(t)\,e^{-\rho}\,\overline{d}_i(t)\frac{\left((w-c)\,e^{r(t,i)}\right)^{1-\gamma}}{1-\gamma} + e^{-\rho}\left(1 - \overline{d}_i(t)\right)\sum_{j=1}^{n} p_{ij}(t)\frac{K_{t+1,j}}{1-\gamma}\left[(w-c)\,e^{r(t,i)}\right]^{1-\gamma}\right\}$$

From the first-order condition we obtain:

$$q_i(t)\,c^{-\gamma} = b(t)e^{r(t,i)-\rho}\overline{d}_i(t)\,e^{-r(t,i)\gamma}(w-c)^{-\gamma} + e^{r(t,i)-\rho}\left(1 - \overline{d}_i(t)\right)e^{-\gamma r(t,i)}\,(w-c)^{-\gamma}\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}$$

Rearranging yields:

$$q_i(t)\,c^{-\gamma} = (w-c)^{-\gamma}\,e^{r(t,i)-\rho}e^{-r(t,i)\gamma}\left[\overline{d}_i(t)\,b(t) + \left(1 - \overline{d}_i(t)\right)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\right]$$

which implies:

$$q_i(t)^{-1/\gamma}\,c = (w-c)\;e^{(\rho - r(t,i))/\gamma}\,e^{r(T,i)}\left[\overline{d}_i(t)\,b(t) + \left(1 - \overline{d}_i(t)\right)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\right]^{-1/\gamma}$$

Rearranging further yields:

$$c = w \times \frac{e^{r(t,i)}\Big[e^{r(t,i)}\Big[\overline{d}_i(t)\,b(t) + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]\Big]^{-1/\gamma}}{e^{\rho}\,q_i(t)^{-1/\gamma} + e^{r(t,i)}\Big[e^{r(t,i)}\Big[\overline{d}_i(t)\,b(t) + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]\Big]^{-1/\gamma}}$$
$$= w \times \underbrace{\left[1 + e^{-r(t,i)}\left(\frac{e^{r(t,i)}\Big[\overline{d}_i(t)\,b(t) + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]}{e^{\rho} q_i(t)}\right)^{\frac{1}{\gamma}}\right]^{-1}}_{c_{t,i}}$$

Thus we obtain:

$$V(t,w,i) = q_i(t)\,c_{t,i}^{1-\gamma}\,\frac{w^{1-\gamma}}{1-\gamma} + b(t)\,e^{-\rho}\,\overline{d}_i(t)\,\frac{w^{1-\gamma}}{1-\gamma}\Big(1-c_{t,i}\Big)^{1-\gamma} e^{r(t,i)(1-\gamma)} + e^{-\rho}\Big(1-\overline{d}_i(t)\Big)\,\frac{w^{1-\gamma}}{1-\gamma}\Big(1-c_{t,i}\Big)^{1-\gamma} e^{r(t,i)(1-\gamma)} \sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}$$
$$= \frac{w^{1-\gamma}}{1-\gamma}\left[q_i(t)\,c_{t,i}^{1-\gamma} + e^{-\rho}\Big(1-c_{t,i}\Big)^{1-\gamma} e^{r(t,i)(1-\gamma)}\left[\overline{d}_i(t)\,b(t) + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\right]\right]$$
$$= \frac{w^{1-\gamma}}{1-\gamma}\,\frac{q_i(t)\,e^{r(t,i)(1-\gamma)}\Big[e^{r(T,i)}\Big(\overline{d}_i(t)\,b(t) + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big)\Big]^{1-1/\gamma} + e^{-\rho}\,e^{r(t,i)(1-\gamma)}(e^{\rho} q_i(t))^{1-1/\gamma}\Big[\overline{d}_i(t)\,b(t) + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]}{\left[(e^{\rho} q_i(t))^{-1/\gamma} + e^{r(t,i)}\Big[e^{r(t,i)}\Big[\overline{d}_i(t)\,b(t) + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]\Big]^{-\frac{1}{\gamma}}\right]^{1-\gamma}}$$
$$= \frac{w^{1-\gamma}}{1-\gamma}\,\frac{e^{r(t,i)(1-\gamma)} q_i(t)\Big[\overline{d}_i(t)\,b(t) + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]}{\left[(e^{\rho} q_i(t))^{-1/\gamma} + e^{r(t,i)}\Big[e^{r(t,i)}\Big[\overline{d}_i(t)\,b(t) + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\Big]\Big]^{-\frac{1}{\gamma}}\right]^{-\gamma}}$$
$$= \frac{w^{1-\gamma}}{1-\gamma}\underbrace{\left[q_i(t)^{\frac{1}{\gamma}} + e^{-r(t,i)}\left[e^{r(t,i)-\rho}\left(\overline{d}_i(t)\,b(t) + \Big(1-\overline{d}_i(t)\Big)\sum_{j=i}^{n} p_{ij}(t)\,K_{t+1,j}\right)\right]^{\frac{1}{\gamma}}\right]^{\gamma}}_{K_{t,i}}$$

■

Proof of Corollary C.2. The proof follows immediately from the expression for VSI, given by equation (8), and from noting that $u_c(c_i(0), q_i(0)) = q_i(0)\,c_{i,0}^{-\gamma}\,w^{-\gamma}$. ■

D Complete Markets Model

We assume a full menu of actuarially fair annuities is available, where consumers can choose consumption streams, $c(t)$, that depend on the evolution of their health state. Thus, the consumer is able to fully insure against consumption risk. The consumer's maximization problem is:

$$\max_{c(t)} \mathbb{E}\left[\int_0^T e^{-\rho t} S(t)\, u(c(t), q_{Y_t}(t))\, dt \middle| Y_0 \right] \tag{D.1}$$

subject to:

$$\mathbb{E}\left[\int_0^T e^{-rt} S(t) c(t)\, dt \middle| Y_0 \right] = W_0 + \mathbb{E}\left[\int_0^T e^{-rt} S(t)\, m_{Y_t}(t)\, dt \middle| Y_0 \right] \equiv \overline{W}(0, Y_0)$$

where $\overline{W}(0, Y_0)$ is the net present value of wealth and future earnings.

The consumer chooses the consumption profile at time t based on her health state, $Y_t = i$, and on her available wealth, $\overline{W}(t,i)$. We define the present value of future earnings as:

$$M(t,i) = \mathbb{E}\left[\int_t^T e^{-r(u-t)} \exp\left\{ -\int_t^u \mu(s)\, ds \right\} m_{Y_u}(u) du \middle| Y_t = i \right]$$

Her available wealth finances future consumption such that:

$$\overline{W}(t,i) = \mathbb{E}\left[\int_t^T e^{-r(u-t)} \exp\left\{ -\int_t^u \mu(s) ds \right\} c(u)\, du \middle| Y_t, \overline{W}(t,i) \right]$$

Lemma D.1. *The law of motion for wealth is:*

$$\frac{\partial \overline{W}(t,i)}{\partial t} = r\overline{W}(t,i) - c\left(t, \overline{W}(t,i), i\right) + \sum_{j>i} \lambda_{ij}(t) \left[\overline{W}(t,i) - \overline{W}(t,j) \right],\ i = 1, \ldots, n,\ \overline{W}(t, n+1) = 0\, \forall t$$

Proof. See Appendix D.1 ■

Note that the dynamics for $\overline{W}(t,i)$ will depend on $\overline{W}(t,j)$, $j > i$, so that $\left(Y_t, \overline{W}(t, Y_t)\right)$ is not Markov, but $\left(Y_t, \overline{W}(t)\right)$, where we define the wealth vector $\overline{W}(t) \equiv \left(\overline{W}(t,1), \ldots, \overline{W}(t, n+1)\right)$, is Markov.

Define the optimal value-to-go function as:

$$V\left(t, \overline{W}(t), Y_t\right) = \max_{c(u)} \mathbb{E}\left[\int_t^T e^{-\rho(u-t)} \exp\left\{ -\int_t^u \mu(s)\, ds \right\} u\left(c(u), q_{Y_u}(u)\right) du \middle| Y_t, \overline{W}(t) \right]$$

subject to the law of motion for wealth given above. As a stochastic dynamic programming problem, $V(\cdot)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\begin{aligned} \rho V(t, \overline{W}(t), i) = \frac{\partial V(t, \overline{W}(t), i)}{\partial t} + \max_{c(t)} \Bigg\{ & u(c(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) \left[V(t, \overline{W}(t), j) - V(t, \overline{W}(t), i) \right] \\ & + \sum_{k \geq i} \frac{\partial V(t, \overline{W}(t), i)}{\partial \overline{W}(t,k)} \left[r\, \overline{W}(t,k) - c(t) + \sum_{l>k} \lambda_{kl}(t) \left[\overline{W}(t,k) - \overline{W}(t,l) \right] \right] \Bigg\},\ 1 \leq i \leq n \end{aligned} \tag{D.2}$$

where $V(t, \overline{W}(t), n+1) = 0$. Similarly to the uninsured case presented in the main text, we follow Parpas and Webster (2013) and focus on the path of Y that begins in state i and remains in i until time t, with $c_i(t)$ and $\overline{W}_i(t)$ denoting the corresponding optimal consumption and wealth paths. We take optimal consumption rules and value functions from other states as exogenous. As in the uninsured case, this approach will allow us to apply the standard Pontryagin maximum principle and derive analytic expressions.

Lemma D.2. *The optimal value function for $Y_0 = i$, $V\left(0, \overline{W}(0,i), i\right)$, for the following deterministic optimization problem also satisfies the HJB given by (D.2), for each $i \in \{1,\dots,n\}$:*

$$V\left(0, \overline{W}(0,i), i\right) = \max_{c_i(t)} \left[\int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V\left(t, \overline{W}_i(t), j\right) \right) dt \right] \tag{D.3}$$

subject to:

$$\frac{\partial \overline{W}_i(t,j)}{\partial t} = r\overline{W}_i(t,j) - c\left(t, \overline{W}_i(t), j\right) + \sum_{k>j} \lambda_{jk}(t) \left[\overline{W}_i(t,j) - \overline{W}_i(t,k) \right], \; j > i$$

$$\frac{\partial \overline{W}_i(t,i)}{\partial t} = r\overline{W}_i(t,i) - c_i(t) + \sum_{k>i} \lambda_{ik}(t) \left[\overline{W}_i(t,i) - \overline{W}_i(t,k) \right]$$

where $V\left(t, \overline{W}_i(t), j\right)$ and $c\left(t, \overline{W}_i(t), j\right)$, $j > i$, are taken as exogenous.

Proof. See Appendix D.1 ■

Following Bertsekas (2005), the Hamiltonian for the (deterministic) maximization problem (D.3) is:

$$\begin{aligned} H\left(\overline{W}_i(t), c_i(t), p_i(t)\right) = e^{-\rho t} \tilde{S}(i,t) & \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V\left(t, \overline{W}_i(t), j\right) \right) \\ & + \sum_{k>i} p_i(t,k) \left[r\overline{W}_i(t,k) - c\left(t, \overline{W}_i(t), k\right) + \sum_{l>k} \lambda_{kl}(t) \left[\overline{W}_i(t,k) - \overline{W}_i(t,l) \right] \right] \\ & + p_i(t,i) \left[r\overline{W}_i(t,k) - c_i(t) + \sum_{l>i} \lambda_{il}(t) \left[\overline{W}_i(t,i) - \overline{W}_i(t,l) \right] \right] \end{aligned} \tag{D.4}$$

where $p_i(t) = (p_i(t,1), \dots, p_i(t,n))$ is the vector of costate variables corresponding to wealth $\overline{W}_i(t)$.

Lemma D.3. *We have that $p_i(t,i) = \theta e^{-\rho t} \tilde{S}(i,t)$ for θ independent of i, and $p_i(t,k) = 0, k \neq i$. The necessary first-order condition for consumption is:*

$$e^{(r-\rho)t} u_c(c_i(t), q_i(t)) = \theta \tag{D.5}$$

where $\theta = p_i(0,i) = \partial V\left(0, \overline{W}_i(0), i\right) / \partial \overline{W}(0,i)$ is the marginal utility of wealth.

Proof. See Appendix D.1 ■

Equation (D.5) shows that the discounted marginal utility of consumption is constant within the path that remains in state i. The following result extends this insight by showing that the same is true across states.

Lemma D.4. *The first-order condition (D.5) holds across different states. That is, if a consumer transitions from state i to state j, then $u_c\left(c(t,i,\overline{W}(t)), q_i(t)\right) = u_c\left(c(t,j,\overline{W}(t)), q_j(t)\right) \; \forall j$.*

Proof. See Appendix D.1 ■

To analyze the value of life, let $\delta_{ij}(t)$, $i < j, i \leq n, j \leq n+1$, be a perturbation on the transition rate $\lambda_{ij}(t)$, where $\sum_{j>i} \int_0^T \delta_{ij}(t) dt = 1$, and consider:

$$\tilde{S}^{\varepsilon}(i,t) = \exp\left[-\int_0^t \sum_{j>i} \left(\lambda_{ij}(s) - \varepsilon \delta_{ij}(s) \right) ds \right], \text{ where } \varepsilon > 0$$

Proposition D.5. *The marginal utility of preventing an illness or death is given by:*

$$\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T \left(\tilde{S}(i,t) \left\{ e^{-\rho t} \left[u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V\left(t, \overline{W}_i(t), j\right) \right] + \theta e^{-rt} \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) \left[\overline{W}_i(t,j) - M(t,j) \right] \right] \right\} - \tilde{S}(i,t) \sum_{j>i} \delta_{ij}(t) \left\{ e^{-\rho t} V\left(t, \overline{W}_i(t), j\right) - \theta e^{-rt} \left[\overline{W}_i(t,j) - M(t,j) \right] \right\} \right) dt \tag{D.6}$$

Proof. See Appendix D.1 ■

To obtain the value of statistical life (VSL), we first set $\delta_{i,N+1}$ equal to the Dirac delta function, and set all other perturbations equal to 0. Dividing the result by the marginal utility of wealth, θ, then yields:

$$VSL = \int_0^T \tilde{S}(i,t) e^{-rt} \left\{ \left[\frac{u(c_i(t), q_i(t))}{u_c(c_i(t), q_i(t))} + \sum_{j>i} \lambda_{ij}(t) \frac{V\left(t, \overline{W}_i(t), j\right)}{\partial V\left(t, \overline{W}_i(t), j\right) / \partial \overline{W}_i(t,j)} \right] + \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) \left[\overline{W}_i(t,j) - M(t,j) \right] \right] \right\} dt \tag{D.7}$$

$$= \frac{V\left(0, \overline{W}_i(0), i\right)}{u_c(c_i(0), q_i(0))} - W_0$$

$$= \mathbb{E}\left[\int_0^T e^{-rt} S(t) v(t) dt \,\middle|\, Y_0 = i \right]$$

where the the value of a one-period change in survival from the perspective of current time is:

$$v(t) = \frac{u(c(t), q_{Y_t}(t))}{u_c\left(c(t), q_{Y_t}(t)\right)} + m_{Y_t}(t) - c_{Y_t}(t)$$

Differentiating the first-order condition (D.5) with respect to t yields like the life-cycle profile of consumption:

$$\frac{\dot{c}_i(t)}{c_i(t)} = \sigma (r - \rho) + \sigma \eta \frac{\dot{q}_i}{q_i} \tag{D.8}$$

Equation (D.8) matches the result one obtains in a setting with a single health state, such as Murphy and Topel (2006).

To analyze the value of prevention, consider a reduction in the transition probability for only one alternative state, j, so that $\delta_{ik}(t) = 0\ \forall k \neq j$. The value of avoiding illness j is then equal to:

$$VSI(i,j) = \int_0^T \tilde{S}(i,t) e^{-rt} \left\{ \left[\frac{u(c_i(t), q_i(t))}{u_c(c_i(t), q_i(t))} + \sum_{j>i} \lambda_{ij}(t) \frac{V\left(t, \overline{W}_i(t), j\right)}{\frac{\partial V\left(t, \overline{W}_i(t), j\right)}{\partial \overline{W}_i(t,j)}} \right] + \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) \left[\overline{W}_i(t,j) - M(t,j) \right] \right] \right\} dt \tag{D.9}$$

$$- \left[\frac{V\left(0, \overline{W}_i(0), j\right)}{\theta} - \left[\overline{W}_i(0,j) - M(0,j) \right] \right]$$

$$= \frac{V\left(0, \overline{W}_i(0), i\right)}{u_c(c_i(0), q_i(0))} - W_0 - \left(\frac{V\left(0, \overline{W}_i(0), j\right)}{u_c(c_i(0), q_i(0))} - \left[\overline{W}_i(0,j) - M(0,j) \right] \right)$$

$$= VSL(i) - VSL\left(j \middle| W_0 = \overline{W}_i(0,j) - M(0,j)\right)$$

Thus, equation (D.9) demonstrates that $VSI(i,j)$ is equal to the difference in VSL for states i and j, with the caveat that VSL in state j uses a measure of wealth evaluated from the perspective of a person in state i. This technicality arises because the value of the consumer's annuity depends on her expected survival. For example, an annuity is worth more to a healthy 65-year-old than it is to a 65-year-old who was just diagnosed with lung cancer.

A constant value per unit of health arises only when the utility of consumption is constant (Bleichrodt and Quiggin, 1999). Inspecting equation (D.8) shows that when markets are complete, consumption will be constant when the rate of time preference equals the interest rate and quality of life is constant.

D.1 Proofs

Proof of Lemma D.1. Available wealth can be written as:

$$\overline{W}(t,i) = \int_t^T \exp\left\{-\int_t^u r + \sum_{j>i} \lambda_{ij}(s)\, ds\right\} \left[c_i(t,u) + \sum_{j>i} \lambda_{ij}(u)\overline{W}_i(u,t,j)\right] du$$

where with a slight abuse of notation, $c_i(t,u)$ and $\overline{W}_i(u,t,j)$ denote the consumption and wealth paths for an individual who is in state i at time t and remains in state i until time u—but jumps to state j at time u for the latter. The result then follows by taking the derivative with respect to t. ■

Proof of Lemma D.2. This proof follows the same logic as the proof of Lemma 1 in Appendix A. Consider the deterministic optimization problem (D.3). Denote the optimal value-to-go function as:

$$\overline{V}\left(t, \overline{W}_i(t), i\right) = \max_{c_i(t)} \left\{ \int_t^T e^{-\rho u} \tilde{S}(i,u) \left(u\left(c_i(u), q_i(u)\right) + \sum_{j>i} \lambda_{ij}(u) V\left(u, \overline{W}_i(u), j\right)\right) du \right\}$$

Setting $\overline{V}\left(t, \overline{W}_i(t), i\right) = e^{-\rho t}\tilde{S}(i,t) V\left(t, \overline{W}_i(t), i\right)$ then demonstrates that $V(\cdot)$ satisfies the HJB (D.2) for i.

■

Proof of Lemma D.3. The costate equations for the Hamiltonian (D.4) are:

$$\dot{p}_i(t,i) = -\left[r + \sum_{j>i} \lambda_{ij}(t)\right] p_i(t,i),$$

$$\dot{p}_i(t,k) = -e^{-\rho t}\tilde{S}(i,t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V\left(t, \overline{W}_i(t), j\right)}{\partial \overline{W}_i(t,k)} + \sum_{k \geq j > i} p_i(t,j) \left(\frac{\partial c\left(t, \overline{W}_i(t), j\right)}{\partial \overline{W}_i(t,k)} + \lambda_{jk}(t) \right) - p_i(t,k)\left[r + \sum_{l>k} \lambda_{kl}(t)\right] + p_i(T,i)\lambda_{ik}(t)$$

for $k > i$. From the first costate equation, we obtain:

$$p_i(t,i) = e^{-rt}\tilde{S}(i,t)\theta$$

Taking first-order conditions in the Hamiltonian (D.4) and plugging this in then yields:

$$u_c(c_i(t), q_i(t)) = \frac{\partial V\left(t, \overline{W}_i(t), i\right)}{\partial \overline{W}_i(t,i)} = e^{(\rho - r)t}\theta$$

To see that this solution works, let θ be constant across states, and set $p_i(t,k) = 0 = \frac{\partial V\left(t, \overline{W}_i(t), i\right)}{\partial \overline{W}_i(t,k)}$. This expression then satisfies the costate equation system across i, k, and t. In particular, for the second equation we obtain:

$$\dot{p}_i(t,k) = -e^{-\rho t}\tilde{S}(i,t)\lambda_{ik}(t) \underbrace{\frac{\partial V\left(t, \overline{W}_i(t), k\right)}{\partial \overline{W}_i(t,k)}}_{e^{(\rho - r)t}\theta} + \lambda_{ik}(t) p_i(t,i) = 0$$

■

Proof of Lemma D.4. With Lemma D.3, the HJB (D.2) takes the form:

$$\rho V\left(t, \overline{W}(T,i), i\right) = \frac{\partial V(t, \overline{W}(t,i), i)}{\partial t} + \max_{c(t)} \left\{ u(c(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) \left[V(t, \overline{W}(t,j), j) - V\left(t, \overline{W}(t,i), i\right) \right] + \frac{\partial V\left(t, \overline{W}(t,i), i\right)}{\partial \overline{W}(t,i)} \left[r\overline{W}(t,i) - c(t) + \sum_{k>i} \lambda_{ik}(t) \left[\overline{W}(t,i) - \overline{W}(t,k) \right] \right] \right\}, \ 1 \le i \le n$$

By taking the first-order condition, we get:

$$u_c\left(c(t), q_i(t)\right) = u_c\left(c(t,i,\overline{W}(t)), q_i(t)\right) = \frac{\partial V(t, \overline{W}(t,i), i)}{\partial \overline{W}(t,i)}$$

Furthermore, differentiating the HJB (D.2) with respect to $\overline{W}(t,j)$, j fixed, we get:

$$\frac{\partial V(t, \overline{W}(t,j), j)}{\partial \overline{W}(t,j)} = \frac{\partial V(t, \overline{W}(t,i), i)}{\partial \overline{W}(t,i)}$$

Combining these last two results completes the proof:

$$u_c\left(c(t,i,\overline{W}(t)), q_i(t)\right) = u_c\left(c(t,j,\overline{W}(t)), q_j(t)\right)$$

■

Proof of Proposition D.5. Starting from equation (D.3), we have:

$$V^{\varepsilon}\left(0, \overline{W}_i(0,i), i\right) = \int_0^T e^{-\rho t} \exp\left\{ -\int_0^t \sum_{j>i} \lambda_{ij}(s) - \varepsilon \sum_{j>i} \delta_{ij}(s)\, ds \right\} \left[u\left(c_i^{\varepsilon}(t), q_i(t)\right) + \sum_{j>i} \left[\lambda_{ij}(t) - \varepsilon \delta_{ij}(t) \right] V\left(t, \overline{W}_i^{\varepsilon}(t), j\right) \right] dt$$

where $c_i^{\varepsilon}(t)$ and $\overline{W}_i^{\varepsilon}(t)$ represent the equilibrium variations in $c_i(t)$ and $\overline{W}_i(t)$ caused by the perturbation, $\delta_{ij}(t)$. Differentiating then yields:

$$\frac{\partial V}{\partial \varepsilon}\bigg|_{\varepsilon=0} = \int_0^T e^{-\rho t} \tilde{S}(i,t) \left[u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V\left(t, \overline{W}_i(t), j\right) \right] \left[\sum_{j>i} \int_0^t \delta_{ij}(s) ds \right] - e^{-\rho t} \tilde{S}(i,t) \sum_{j>i} \delta_{ij}(t) V\left(t, \overline{W}_i(t), j\right) + e^{-\rho t} \tilde{S}(i,t) \left[\underbrace{u_c(c_i(t), q_i(t))}_{e^{-(r-\rho)t}\theta} \frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\bigg|_{\varepsilon=0} + \sum_{j>i} \lambda_{ij}(t) \underbrace{\frac{\partial V\left(t, \overline{W}_i(t), j\right)}{\partial \overline{W}_i(t,j)}}_{e^{-(r-\rho)t}\theta} \frac{\partial \overline{W}_i(t,j)}{\partial \varepsilon}\bigg|_{\varepsilon=0} \right] dt$$

We have:

$$\begin{aligned}
W_0 &= \mathbb{E}\left[\int_0^T e^{-rt} S(t) \left[c(t) - m_{Y_t}(t) \right] dt \,\middle|\, Y_0 = i \right] \\
&= \int_0^T e^{-rt - \int_0^t \sum_{j>i} \lambda_{ij}(s)\, ds} (c_i(t) - m_i(t))\, dt + \sum_{j>i} e^{-rt - \int_0^t \sum_{j>i} \lambda_{ij}(s)\, ds} \lambda_{ij}(t) \underbrace{\mathbb{E}\left[\int_t^T e^{-r(u-t)} \exp\left\{ -\int_t^u \mu(s)\, ds \right\} c(u)\, du \,\middle|\, Y_t = j \right]}_{\overline{W}_i(t,j)} \\
&\quad - \sum_{j>i} e^{-rt - \int_0^t \sum_{j>i} \lambda_{ij}(s)\, ds} \lambda_{ij}(t) \underbrace{\mathbb{E}\left[\int_t^T e^{-r(u-t)} \exp\left\{ -\int_t^u \mu(s)\, ds \right\} m_{Y_t}(u)\, du \,\middle|\, Y_t = j \right]}_{M(t,j)} \\
&= \int_0^T e^{-rt - \int_0^t \sum_{j>i} \lambda_{ij}(s) ds} \left[c_i(t) - m_i(t) + \sum_{j>i} \lambda_{ij}(t) \left(\overline{W}_i(t,j) - M(t,j) \right) \right] dt
\end{aligned}$$

The budget constraint then implies:

$$
\begin{aligned}
0 &= \left.\frac{\partial W_0}{\partial \varepsilon}\right|_{\varepsilon=0} \\
&= \frac{\partial}{\partial \varepsilon}\int_0^T e^{-rt}\exp\left\{-\int_0^t \sum_{j>i}\lambda_{ij}(s) - \varepsilon\sum_{j>i}\delta_{ij}(s)\,ds\right\}\left(c_i^{\varepsilon}(t) - m_i(t) + \sum_{j>i}\left[\lambda_{ij}(t) - \varepsilon\delta_{ij}(t)\right]\left(\overline{W}_i^{\varepsilon}(t,j) - M(t,j)\right)\right)dt\Bigg|_{\varepsilon=0} \\
&= \int_0^T \Bigg(e^{-rt}\tilde{S}(i,t)\left[c_i(t) - m_i(t) + \sum_{j>i}\lambda_{ij}(t)\left[\overline{W}_i(t,j) - M(t,j)\right]\right]\left[\sum_{j>i}\int_0^t \delta_{ij}(s)ds\right] \\
&\qquad - e^{-rt}\tilde{S}(i,t)\sum_{j>i}\delta_{ij}(t)\left[\overline{W}_i(t,j) - M(t,j)\right] + e^{-rt}\tilde{S}(i,t)\left[\left.\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\right|_{\varepsilon=0} + \sum_{j>i}\lambda_{ij}(t)\left.\frac{\partial \overline{W}_i^{\varepsilon}(t,j)}{\partial \varepsilon}\right|_{\varepsilon=0}\right]\Bigg)dt
\end{aligned}
$$

Plugging this last result into the expression for $\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0}$ then yields the desired result for marginal utility:

$$
\begin{aligned}
\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T \Bigg(\tilde{S}(i,t)\left[\sum_{j>i}\int_0^t \delta_{ij}(s)ds\right]&\left\{e^{-\rho t}\left[u(c_i(t),q_i(t)) + \sum_{j>i}\lambda_{ij}(t)V\left(t,\overline{W}_i(t,j),j\right)\right] + \theta e^{-rt}\left[m_i(t) - c_i(t) - \sum_{j>i}\lambda_{ij}(t)\left[\overline{W}_i(t,j) - M(t,j)\right]\right]\right\} \\
&- \tilde{S}(i,t)\left\{e^{-\rho t}\sum_{j>i}\delta_{ij}(t)V\left(t,\overline{W}_i(t),j\right) - \theta e^{-rt}\sum_{j>i}\delta_{ij}(t)\left[\overline{W}_i(t,j) - M(t,j)\right]\right\}\Bigg)dt
\end{aligned}
$$

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